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THE EFFECTS OF FLIGHT HOURS AND SORTIES
ON
FAILURE RATES

by

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The Effects of Flight Hours and Sorties on Failure Rates

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This thesis focuses on the modeling of F-14A component failure rates. Current methodology employs the Exponential distribution to model component failures and the associated Poisson distribution to determine expected demand. Three other failure rate distributions are explored as alternatives: a Weibull flight hour model, a Geometric sortie-dependent model, and a Mixed sortie-flight hour model. The expected number of component failures is calculated for each model and a comparison is made between the current model and these alternatives. The specific results pertain to aircraft of this type but the concepts employed can be applied to other aircraft as well.

The Geometric model provided a better fit for components which were not operated continuously, and the Weibull performed better when the components were operated continuously. Overall, the Exponential was the least effective model for the nine components studied.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

A. BACKGROUND

1. A Carrier Navy

The carrier is the nucleus of today's U.S. Navy, but its ability to project power is a byproduct of the synergistic relationship it shares with its air wing. As Peter Garrison stated in *CV: Carrier Aviation*, "by adopting submarine-launched ballistic missiles and *ship-based tactical aircraft* the navy has co-opted some of the tools of the army and air force and adapted them to a marine environment. *Ships in themselves are no longer powerful*, they take their power from the airplanes or long range missiles that they carry. An adversary setting out to sink a ship does not send another ship against it, but instead sends an airplane or a missile" [Ref. 1]. (italics mine). The fleet has become dependent upon aircraft to take the battle over the horizon, and to extend the battle group's area of influence. Without downplaying the importance of the carrier, it is just another large ship without its contingent of aircraft. Thus, the carrier's real importance to the fleet rests on the ability of its personnel to support and maintain the air wing at an optimal level of readiness.

2. Aircraft Readiness

Ideally, the fleet would like each aircraft to be fully mission capable (FMC); that is, the material condition of the aircraft should satisfy minimum requirements to perform all of its missions. An aircraft's mission status (e.g., partially mission capable (PMC), not mission capable (NMC), etc.) is determined using the Mission Essential Subsystems Matrices published in OPNAVINST 5442.4H.

The ability to ensure a high percentage of FMC aircraft is constrained by several factors. First, the Navy has not been able to provide shipboard intermediate maintenance activities (IMA) with sufficient numbers of qualified personnel, especially in critical technical billets. In part, this results from the allocation of manpower based on squadron size (i.e., the smaller squadrons have fewer personnel to maintain their aircraft). The problem associated with the manpower shortage is compounded by a need for specialization in troubleshooting and repairing the different aircraft types, models, and series deployed on the carrier. Second, the IMA's capability to repair components as they fail is constrained by limitations of test equipment, maintenance

facility size, availability of repair parts, and the total demand within the repair pipeline. Finally, those components which cannot be fixed at the intermediate level must be resupplied by outside sources.

The air wing must be able to accurately identify and communicate its requirements for component replacements and resupply facilities should be capable of expeditiously responding to the those needs. During periods of high tempo flight operations, as would be experienced during wartime, the factors listed above become more critical. The ability to maintain high levels of readiness is hampered by an increase in the number of components in the repair pipeline, manpower and facility limitations, and a reduced or nonexistent ability to resupply vital components. Potential problems may be partially rectified by improving the methodologies currently used to attain that "best" possible mix of replacement parts located on the aircraft carrier. [Ref. 2: pp. 3,9]

B. AVIATION CONSOLIDATED ALLOWANCES

1. Definition

The Aviation Consolidated Allowance List (AVCAL) is supposed to be a "best" mix inventory. An AVCAL is defined in OPNAVINST 4790.2C as a list of aeronautical material tailored to each individual ship to support assigned or embarked aircraft flight operations. This list of parts is prepared by the Navy's Aviation Supply Office (ASO) under the direction of the aircraft type commander, and determines spare part allowances based on expected demands.

Ideally, it would be advantageous to stock at least one spare for each part that could possibly fail during deployment regardless of how unlikely that occurrence might be. Unfortunately, inventory sizes are constrained by both budget considerations and storage limitations. The latter is the result of a diverse collection of aircraft types, manufactured by different companies with each aircraft requiring its own specific replacement parts, coupled with the inherent aversion to building aircraft carriers even larger than they already are simply to carry more parts. Table 1 lists the normal complement of aircraft aboard a carrier, and is provided to illustrate the differences in aircraft types, missions, and spare part requirements. [Ref. 2: p.5].

2. Need for Improvement

Determination of the proper AVCAL allowance for each component is not an easy task, and is definitely an area needing improvement. A recent study conducted by

TABLE 1
NORMAL COMPLEMENT OF CARRIER AIRCRAFT

AIRCRAFT	MISSION	SQUADRONS	AIRCRAFT
F-14A	Fighter	2	24
A-7E	Light Attack	2	24
A-6E	Attack	1	10
KA-6D	Tanker	1	4
S-3A	ASW	1	10
E-2C	SURV	1	4
EA-6B	EW	1	4
SH-3	ASW	1	6
TOTAL			81

the Center for Naval Analyses reported that 'statistics for combat aircraft operating during the period July 1982 through June 1983 show aircraft operating at less than 60 percent of their anticipated wartime rate. These aircraft were not FMC due to supply between 18-24 percent of the time. These readiness rates would drop significantly if the aircraft flew at a wartime rate and AVCALs continued to be constructed as they were in this period" [Ref. 3: p.1-1]. This statement strengthens the argument that current methods of AVCAL formulation are, at best, adequate for peacetime operations, but could result in disastrously low readiness rates when exposed to the additional requirements associated with wartime utilization. Two approaches to determining a better AVCAL would be the introduction of new methods of modelling requirements or the improvement of weak areas within the current model.

3. AVCAL Models

Two categories of models used to compute the AVCAL are readiness-based and demand-based inventory systems. Readiness-based systems determine spare allowances that achieve a predetermined level of readiness while minimizing the cost of the AVCAL. Examples of readiness-based models are Rand's DYNAMETRIC model and the Center for Naval Analyses' MIME model [Ref. 3: p.2-8]. Demand-based

(supply effectiveness) models determine component allowances based on expected demand and the capabilities of the repair facility and resupply pipeline. The ASO Manual model is an example of a demand-based method, and is the model currently used by the Navy to compute AVCAL spare allowances.

C. ASO MANUAL MODEL

The ASO Manual Model uses historic data to forecast demand rates and component repair times. The model determines the type and number of components included in the AVCAL allowance. For a detailed explanation of the rules governing component selection, the reader should refer to either *Aviation Parts Allowance Policy* by Peter Evanovich or *A Retail Inventory Model for Naval Aviation Repairable Items* by Mark L. Mitchell. [Refs. 3,4].

1. Attrition Only Items

Attrition only items are repairable components that cannot be repaired at the intermediate maintenance activity aboard the carrier. The expected demand for items of this type is computed using equation 1.1.

$$NB' = NB \times (FH'/FH) \qquad \text{(eqn 1.1)}$$

where,

NB' = number of expected attritions for a specific flight program

NB = number of attritions in the ASO data base

FH' = specified flight program

FH = total flight hours represented by the ASO data base

2. Items that can be Repaired Locally

The allowance for items that can be repaired locally is composed of two parts. An allowance for repairable components that are beyond the capabilities of the local maintenance activity to repair them (BCM), and an allowance for components that are queued at various places in the local repair cycle.

a. BCM Allowance

Units are considered BCM if the maintenance activity cannot repair the item because of a lack of tools, equipment, parts or technical expertise. Equation 1.1 is used to forecast the number of BCM failures expected to occur for a specific flight program.

b. Repair Pipeline Allowance

The second input to the local repairable allowance provides units to compensate for the number of failed components undergoing repair at the IMA. This allowance, called the Local Repair Cycle Allowance (LRCA), is set at a quantity which should ensure that demand does not exceed available supply at least ninety percent of the time. This safety level is dependent upon an accurate estimate of the number of components repaired by the shipboard intermediate maintenance activity. Such a forecast is expressed in equation 1.2.

$$NR' = NR \times (FH'/FH) \qquad \text{(eqn 1.2)}$$

where,

NR' = number of expected repairs for a specific flight program

NR = number of repairs represented in the ASO data base

and,

FH' and FH remain as defined above

D. FAILURE RATE DISTRIBUTION

Combining equations 1.1 and 1.2 provides the expected number of total failures, both repairables and attritables, for the specified flight program.[1]

$$NR' + NB' = (NR + NB) \times (FH'/FH) \qquad \text{(eqn 1.4)}$$

Substituting NF' for $NR' + NB'$ and NF for $NR + NB$, equation 1.4 can be expressed as:

$$NF' = NF \times (FH'/FH) \qquad \text{(eqn 1.5)}$$

Equation 1.6 results from rearranging terms on the right hand side of equation 1.5.

$$NF' = (NF/FH) \times FH' \qquad \text{(eqn 1.6)}$$

[1] For "attrition only items" NR' equals zero.

The total number of expected failures NF' equals the product of some specific future flight program (FH') and the quantity NF/FH. The latter should be easily recognized as the maximum likelihood estimate (based on the historical data base) for the failure rate lambda. Thus, NF' is the expected value of a Poisson distribution with the failure rate parameter λ and interval of observation (0,FH']. The use of the Poisson distribution implies that the underlying distribution for the time between failures is exponential. Certain properties of the exponential distribution simplify the computational problem, but what price is paid for this simplicity? The remainder of this paper will suggest alternative distributions for describing the time between component failures and estimating the expected number of failures for a given flight program.

E. SYNOPSIS

Chapter II describes the methods and assumptions used to extract component failure information from a data base containing six squadrons of F-14A aircraft flight and maintenance records. Chapters III and IV discuss current and possible methods of modeling aircraft failure rates, and techniques used to derive maximum likelihood estimates of model parameters. Flight hour dependent models are presented in Chapter III and include the traditional Exponential model and a Weibull model. Chapter IV discusses a sortie-dependent Geometric model, and the sortie-flight hour Mixed model proposed in *Distinguishing the Effects on Failures of Changes in Sortie Rate and Sortie Length* by Robert A. Levy [Ref. 5]. The four different models will be compared in Chapter V using both graphical and quantitative measures of fit. Finally, Chapter VI will discuss the advantages and disadvantages of each model, along with the conclusions and recommendations.

II. DATA BASE

A. ORIGIN OF DATA

The initial data base was obtained from the Navy Maintenance Support Office (NMSO), Mechanicsburg, Pennsylvania via the Center for Naval Analyses, Alexandria, Virginia. The data set included the historical flight and maintenance records for two operational deployments of F-14A aircraft. The most recent deployment included two squadrons, VF-114 and VF-213, which operated off the USS ENTERPRISE (CVN-65) during the period of March 1984 through December 1984. This deployment became the "model" set, with its flight and maintenance records providing the parameter estimates for the study.

The second, earlier deployment consisted of four squadrons, VF-11/VF-31 and VF-14/VF-32, which operated off the USS KENNEDY (CV-67) and USS INDEPENDENCE (CV-62), respectively, during the period of November 1983 through March 1984. In this study these four squadrons were treated as the "validation" set.

Using the parameter estimates from the model set, four specific failure rate distributions were used to calculate the expected number of component failures which should occur during a deployment similar to the one flown by the four squadrons aboard the USS KENNEDY and USS INDEPENDENCE. The actual number of component failures observed during the deployment of these squadrons was then compared with the expected number generated using the parameters estimated from the "model" set deployment.

B. MAINTENANCE DATA SYSTEM (MDS)

1. Purpose

The maintenance data system (MDS) was developed to improve the management capabilities of the Navy's Maintenance and Material Management (3-M) system. It was designed as a management information system which would provide the statistical data needed to analyze:

- Equipment maintainability and reliability
- Equipment configuration to include alterations and technical directive compliance
- Equipment mission capability and utilization rates
- Material usage

- Maintenance and material procurement times
- Weapon system and material costing

To meet the data requirements listed above, NMSO divides the data collection effort into four specific areas: material reporting (MR), subsystem capability impact reporting (SCIR), maintenance data reporting (MDR), and utilization reporting. The material reporting system documents those materials used in support of the maintenance efforts. The subsystem capability impact report provides the higher level commanders with information pertaining to their subordinate command's ability to conduct their mission. The last two areas of data collection, maintenance data reporting and utilization reporting, provided the data necessary to determine the times between component failures.

2. Maintenance Data Reporting (MDR)

The documents which support the maintenance data reporting system include support action forms (SAF), metrology equipment recall (METER) cards, and the Visual Information Display System/Maintenance Action Forms (VIDS/MAF). Support action forms document maintenance actions that did not require any corrective action, such as, aircraft servicing, aircraft handling, and the preliminary look phase of an inspection. METER cards identify the maintenance time spent calibrating and repairing the test and measuring systems. The last form, VIDS/MAF (OPNAV 4790/60), documents inspections, technical directive compliance, and repair actions. It identifies the component failures by work unit code, time of failure (julian date and when discovered code), aircraft experiencing the failure (bureau number), cause of failure (malfunction description code), and type of maintenance required (transaction code and action taken code).

3. Utilization Reporting

The flight records (utilization rates) were compiled from information recorded on the Aircraft Yellow Sheet (OPNAV 3760/2B). The Yellow Sheet is prepared by air crew personnel at the end of each mission and contains information identifying the flight according to aircraft flown, mission type, julian date, flight duration, etc.

C. SELECTED WORK UNIT CODES

A work unit code (WUC) is an alphanumeric code identifying an item on which work is being performed. Systems are identified with two digit codes and components with five digit codes. Seven digit codes are used to further breakdown components into

lower levels of subassembly. The components selected for this study are listed in Table 2. Each of the nine components chosen had at least thirty failures during the deployment cycle.

TABLE 2

COMPONENTS WITH ASSOCIATED WORK UNIT CODES

COMPONENT	WUC
CN1263/ASN92(V) INERTIAL MEASURING UNIT	734H100
ECU74/A SIGNAL DATA CONVERTER	46X1600
MXU611/A FUEL TANK RELEASE MECHANISM	4622100
CP1166/A AIR DATA COMPUTER	56X2500
AN/ARC159/159(V)-1 UHF TRANSMITTER	632Z100
T1224/AWG9 RADAR TRANSMITTER	74A1500
IP1185/AWG9 DETAIL DATA DISPLAY	74A5M00
CP1248/ASW43 AIRCRAFT ROLL COMPUTER	5772200
IP1027/AVA12 ANALOG DISPLAY INDICATOR	6918100

D. DATA REDUCTION

The intent of the data reduction process is to eliminate those maintenance actions which do not identify a component failure (e.g., aircraft servicing, troubleshooting, calibrations, or other routine maintenance actions). The maintenance codes not indicative of a component/item failure are found within the series of malfunction description, transaction, and action taken codes. [Ref. 6: pp. 17-19]

1. Malfunction Description Codes

The three digit numeric malfunction description codes describe, as accurately as possible, the probable cause of a system/component failure. Malfunction description codes used when no failure was detected are displayed in Table 3. Maintenance records which contained these codes were eliminated from the final data set. Conditional malfunction description codes identify component failures caused by factors other than natural wear-out. Examples of this type of failure include: failed due

to weather, foreign object damage (FOD), improper or faulty maintenance, etc. To reduce the complexity of parameter estimation, conditional failures were treated as if the failure had occurred through normal usage. The components studied had a low percentage of conditional failures. In fact, less than three percent of all observed failures were conditional.

TABLE 3

EXCLUDED MALFUNCTION DESCRIPTION CODES

CODE	DESCRIPTION
799	No defect
800	No defect - removed and/or reinstalled to facilitate maintenance
801	No defect - removed for modification
803	No defect - removed for time change
804	No defect - removed for scheduled maintenance
805	No defect - removed for pool stock
806	No defect - removed as part of a matched system
807	No defect - removal directed by higher authority
811	No defect - removed during troubleshooting

2. **Transaction Codes**

Transaction codes are two digit numeric codes used to identify the type of data reported by the maintenance activity. Transaction codes used in the maintenance reporting system are listed in Appendix A. Table 4 lists those transaction codes indicative of maintenance actions that required removal, installation, and/or repair of a defective component. Maintenance records associated with these codes were included in the final data set. All other maintenance records were deleted.

3. **Action Taken Codes**

Action taken codes are one digit alphanumeric codes which describe the maintenance action performed on the system, component or item. A complete list of action taken codes is shown in Appendix B. Action taken codes associated with the removal, installation, and/or repair of a defective component were included in the final data set and are shown in Table 5.

TABLE 4

TRANSACTION CODES USED IN STUDY

CODE	DESCRIPTION
12	On-equipment work, including engines, involving non-repairable components/items documented as failed parts.
21	Will be used when a repairable component is removed, excluding engine components, for processing at an I or D level maintenance activity (This code is used when only the removal must be documented and a replacement is not required).
23	Removal and replacement of a defective or suspected defective component from an end item, excluding engines at the O-level. Additionally, this code is used for the removal and replacement of a complete engine assembly for a defect, suspected defect, or scheduled maintenance requirement.
24	Will be used when a repairable engine component is removed for processing at an I or D level activity (This code is used when only the removal must be documented and a replacement is not required).
25	Removal and replacement of a defective or suspected defective repairable component from an engine.

E. FINAL DATA SET

1. Problems

Several problems were encountered during the data reduction process that require discussion. First, it was not always clear which flight evolution caused a component to fail. The cause for this confusion is the poor record keeping associated with the actual time of component failures. For example, the maintenance data reporting system does not record the actual time of component failure. Instead, the data is arranged according to the time the maintenance activity received the component for repair. In most, but not all cases, the component was received by the maintenance activity immediately following the flight causing the component failure.

Second, several records indicated multiple failures of a single component. For example, a single component may experience several unrelated malfunctions during a single flight evolution. To expedite the repair and or replacement of the failed component, the maintenance reporting system requires the documentation of each

TABLE 5
ACTION TAKEN CODES

CODE	DESCRIPTION
1	BCM - maintenance activity not authorized to conduct repair
2	BCM - Lack of equipment, tools, or facilities
3	BCM - Lack of technical skills necessary to complete repair
4	BCM - Lack of parts
5	BCM - Fails check and test, and maintenance is allowed to conduct check and test only
6	BCM - Lack of technical data
7	BCM - Beyond the authorized repair depth
8	BCM - Administrative
9	BCM - Condemned, repair not feasible
C	This code is used when a repairable item of material identified by a work unit code is repaired.
F	Failure of components/items undergoing test.
P	This code is entered when an item of material is removed and only the removal is to be accounted for.
Q	This code is entered when an item is installed and only the installation action is to be accounted for.
R	This code is entered when an item of material is removed due to suspected malfunction and the same or a like item is reinstalled.

* BCM: Action taken codes labelled 1-9 are used for repairable items which have been administratively and or technically screened and found to be beyond the capability of the maintenance activity.

malfunction (i.e., multiple failures of a single component). Previous studies have treated this problem as independent single component failures with identical interoccurrence times. The alternative is to treat the multiple failure as a single incident.

Third, the underlying assumption of the exponential distribution is that components fail as a function of flight hours flown. This would imply that a component is operated continuously, which may or may not be the case.

Fourth, it was difficult to determine if a component was replaced immediately following the flight evolution causing the failure. Additionally, there was some concern pertaining to periods of flight operations during which a component might be missing.

Fifth, the condition of the component (i.e., new or used) at the start of deployment was not always known. A related problem involves the use of the same failure rate distribution for both repaired and new components.

2. Simplifying Assumptions

Before proceeding with any data analysis, it is necessary to make assumptions which can simplify or eliminate the problems discussed in the previous section. These assumptions are presented in the same order as the problem areas they are intended to simplify.

First, all failures are associated with the sortie during which they occurred for When Discovered Codes A, B, C, and D. For all other codes, it is assumed that the training and experience of the flight and ground crews ensure the discovery of the failure at the earliest possible opportunity (i.e., the failure occurred during the flight prior to discovery). A complete listing of When Discovered Codes is found in Appendix C.

Second, all multiple failures of a single component are treated as a single component failure. It is believed that multiple failures would affect the turnaround time required to repair the component and may cause delays within the repair pipeline. However, this should not affect the rate at which the component fails.

Third, it is assumed that components operate continuously, at least in a standby mode. If this is not the case, the sortie-dependent models should provide a better estimate of expected failures.

Fourth, the study assumes that components are replaced immediately and that gaps in coverage are not allowed. In fact, it is extremely unlikely that an aircraft would operate without any of the components listed in this thesis.

Fifth, all components are considered new at the start of the deployment. For the Exponential and Geometric models this assumption is not necessary due to the "memoryless property" they exhibit. Additionally, the impact on the Weibull distribution should be minimal if the number of observed failures is large.

Finally, to reduce the complexity of the problem, it is necessary to assume that repaired components exhibit the same failure characteristics as new components (i.e., have the same failure rate).

3. Format

a. Parameter Estimation

To obtain the best possible estimate of the time between failures, the observations within the initial data base were sorted by aircraft bureau number, then Julian date and the event time. Failure times were measured between consecutive component failures on a specific aircraft. These measurements were never made between component failures on two different aircraft. The information included in the final data set is the aggregate totals and includes a censoring indicator, number of sorties flown, flight time without failure, and flight time with failure.

A censor indicator equal to zero indicates that a component failure was observed. The data entries associated with this type of observation include the number of flights flown inclusive of the event causing the failure, total flight time flown since last failure (does not include flight time of event causing failure), and the flight time of the event causing the failure.

A censor indicator equal to one represents an event which is right hand censored. Right hand censoring refers to an observation of a component which does not fail prior to the termination of the deployment (i.e., the data set documents a component's survival time, but provides no information pertaining to the actual time of component failure). The data entries associated with this type of observation are the total number of flights flown and the total flight time since the last component failure.

b. Model Comparison

The intent of this thesis is to determine each model's ability to predict failures for the components listed, and compare those predictions with the actual component failures observed during the deployment of the squadrons aboard the USS KENNEDY and USS INDEPENDENCE. To accurately assess these capabilities, it was necessary to identify component failures as they occurred during the deployment. Thus, the data set used to provide the graphical and quantitative comparison of the predicted vs actual component failures was different than the data set used to estimate the model parameters. The latter data set was sorted by Julian date and event time only. The accumulated number of sorties flown and flight hour totals were recorded with the observation of each failure, such that, comparisons could be made with the expected failures generated by the failure rate models.

4. Summary Statistics

Table 6 summarizes the two data sets and provides such information as the number of aircraft assigned, sorties flown, flight hour totals, component failures, etc.

TABLE 6
SUMMARY STATISTICS

DATA SET	AIRCRAFT	SORTIES	FLT HRS	FLT HRS/SORTIE MEAN	FLT HRS/SORTIE STD.DEV.
MODEL	30	3489	7608.8	2.18	.98
VALIDATION	44	3012	7030.4	2.33	.54

WUC	MODEL SET FAILURES	VALIDATION SET FAILURES
734H100	105	58
46X1600	34	49
4622100	49	6
56X2500	85	79
632Z100	49	57
74A1500	107	75
74A5M00	118	104
5772200	34	33
6918100	81	93

MODEL SET = VF-114 and VF-213

VALIDATION SET = VF-11, VF-14, VF-31, and VF-32

III. FLIGHT HOUR MODELS

A. OVERVIEW

The two models discussed in this chapter describe component failures as a function of flight hours only. The first flight hour model to be examined is the exponential, which is the model currently used by the Navy to describe the distribution of the time between component failures. This model has some weaknesses inherited from the properties of the exponential distribution. The effect these problem areas might have on the model's ability to accurately predict component failures will be discussed in the next section. The three models offered as alternatives should provide improvement to some or all of these problem areas, and the comparisons made in Chapter V should shed new light on the magnitude of these deficiencies.

The second flight hour model discussed is the Weibull. It is offered as an alternative method of describing the time between failures when flight hours are the only factors contributing to the component malfunction.

B. EXPONENTIAL MODEL

1. Weaknesses

There are two major weaknesses associated with this model that cause some concern. First, the exponential is a flight hour model and as such it is insensitive to the effects of the sortie on the component's time of failure. In Chapter V it will be shown that the expected number of component failures for the time interval $(0,t]$ is proportional to t, so that the expected number of failures for a specific flight hour total is not affected by the average sortie length (i.e., the number of sorties flown).

Second, the exponential distributions are "NO WEAR" distributions, which indicates that a component's probability of surviving a flight of duration t is the same for new components as it is for components which are "S" hours old (i.e., the conditional survival probability is equal to the unconditional probability of survival). This is the "memoryless" property of the exponential distribution, and implies that the prior use of a component has no effect on the time of its failure.

The same concern is reflected in the exponential distribution's constant hazard rate $r(t)$ (i.e., $r(t) = \lambda$ for all values of t). As such, the failure rate λ would not be affected by the time on test t. It is believed that certain components exhibit hazard

rates that are decreasing functions of time (t). In fact, many pieces of electronic equipment are routinely exposed to burn-in periods to make the component less susceptible to failure when put into operation. Conversely, it can be argued that other components have hazard rates that increase with time (i.e., a component is more likely to fail as it experiences some wear and tear). The exponential distribution is not capable of modelling these differences.

2. Maximum Likelihood Results

The maximum likelihood estimate, λ', of the exponential failure rate parameter λ (failures/hour) is found by iteratively solving equation 3.1:[2]

$$\sum_i T_i = \sum_j \frac{t_j \exp(-\lambda t_j)}{1-\exp(-\lambda t_j)} \qquad T_i, t_j \geq 0,\ \lambda > 0,\ m < n \quad i = 1,2,\ldots,n \quad j = 1,2,\ldots,m \qquad \text{(eqn 3.1)}$$

where, the left hand side of equation 3.1 equals the aggregate flight time for those sorties that did not experience a specific component failure, t_j is the duration of the sortie causing the failure, n is the total number of records, censored and uncensored, within the final data set, and m is the number of uncensored observations (failures).

If the sortie lengths, t_j, are equal to t for all j, then equation 3.1 can be simplified to the closed form expression given in equation 3.2. Thus, the average sortie length can be used to provide a "rough" estimate of the maximum likelihood estimator λ':

$$\lambda' = \frac{-\ln(FTWF/TFT)}{t} \qquad \text{(eqn 3.2)}$$

where, FTWF (Flight Time Without Failure) equals the sum of the T_i's for $i = 1,\ldots,n$, TFT equals the total flight time for the deployment, and t equals the average sortie length.

The maximum likelihood estimate for λ' was found using the FORTRAN program listed in Appendix E. The parameter estimates and 95 percent confidence limits for these estimates are displayed in Table 7.

[2] For the derivation of this maximum likelihood equation, the reader is referred to Appendix D.

TABLE 7
EXPONENTIAL MAXIMUM LIKELIHOOD RESULTS

WUC	λ'	σ_{λ}'	CL_{λ}
734H100	.0140	.0018	(.0106,.0175)
46X1600	.0048	.0010	(.0029,.0067)
4622100	.0065	.0011	(.0043,.0086)
56X2500	.0114	.0021	(.0073,.0155)
632Z100	.0067	.0012	(.0043,.0090)
74A1500	.0143	.0019	(.0105,.0181)
74A5M00	.0159	.0024	(.0113,.0206)
5772200	.0048	.0011	(.0026,.0070)
6918100	.0108	.0015	(.0079,.0137)

*** CL = 95 PERCENT CONFIDENCE LIMITS

3. Quantile-Quantile Plots

A graphical method often used to determine an empirical data's goodness of fit with respect to a theoretical distribution is the quantile-quantile plot. The technique requires the observations, $X_1, X_2, \ldots, X_n$, to be ordered from smallest to largest. Empirical quantiles are then computed for these order statistics based on a formula, such as $Q_i = (i-.5)/n$, where Q_i is the empirical quantile for the $X_{(i)}$ order statistic and n equals the total number of observations. These empirical quantiles are plotted against the theoretical quantiles to produce a graphical estimate of goodness of fit.

Some adjustments to the above technique must be made to overcome the problems of censoring. The first adjustment relates to the problem associated with incomplete knowledge of the actual time of component failure. In Chapter II, it was mentioned that the failure occurred during the interval T_i to $T_i + t_i$. To simplify this problem, it is assumed that the midpoint of that interval represents the actual time of failure.

The second adjustment that needs to be made accounts for those observations that are right hand censored. The Kaplan-Meier method for quantile estimation will account for those observations and will provide a more accurate estimate for the empirical quantiles. This method also requires that each observation, censored and uncensored, be ordered from smallest to largest. The empirical quantiles (Q_i) are computed using equation 3.3:

$$Q_i = 1 - [(n+.5)/n] \prod_{\substack{k \text{ in } I \\ k \leq i}} [(n-k+.5)/(n-k+1.5)] \quad \text{for } i \text{ in } I \qquad \text{(eqn 3.3)}$$

where, I is the set of all uncensored observations, and n is the total number of observations, censored and uncensored. [Ref. 7: p. 234]

An exponential quantile-quantile plot was constructed for each of the nine components studied. The quantile-quantile plots for work unit codes 734H100 and 56X2500 are displayed in Figures 3.1 and 3.2, and the remaining quantile-quantile plots are presented in Appendix F. The y-axis of the plot represents the quantiles, $P(T \leq t)$, and the x-axis represents the time between component failures. The asterisks identify the uncensored observations, the ×'s identify the locations of the censored observations, and the theoretical distribution evaluated using λ' is defined by the solid line. The empirical data is fit well by the theoretical distribution if the data points are tightly packed around the solid line. Data points which deviate from the line can provide information about areas of weakness.

Figure 3.1 illustrates the best exponential fit of all the quantile-quantile plots, but the plot still shows the lower to mid quantiles associated with the empirical data points overestimate the quantiles computed using the theoretical distribution. Figure 3.2 provides a better representation of the other quantile-quantile plots. In every plot, the empirical quantiles overestimate the theoretical quantiles for quantiles less than .5, and underestimate the theoretical quantile for quantiles greater than .75.

Figure 3.3 displays the Weibull cumulative distribution function for α equal to .5, 1, and 2. The reader should be aware that a Weibull distribution with shape parameter (α) equal to 1 is the exponential distribution. Figure 3.3 shows that for values of time less than t_0, the cumulative distribution for the curve represented by $\alpha = .5$ is greater than the exponential curve ($\alpha = 1$), and for values of time greater than t_0 it is less than the cumulative distribution of the exponential curve. This is the

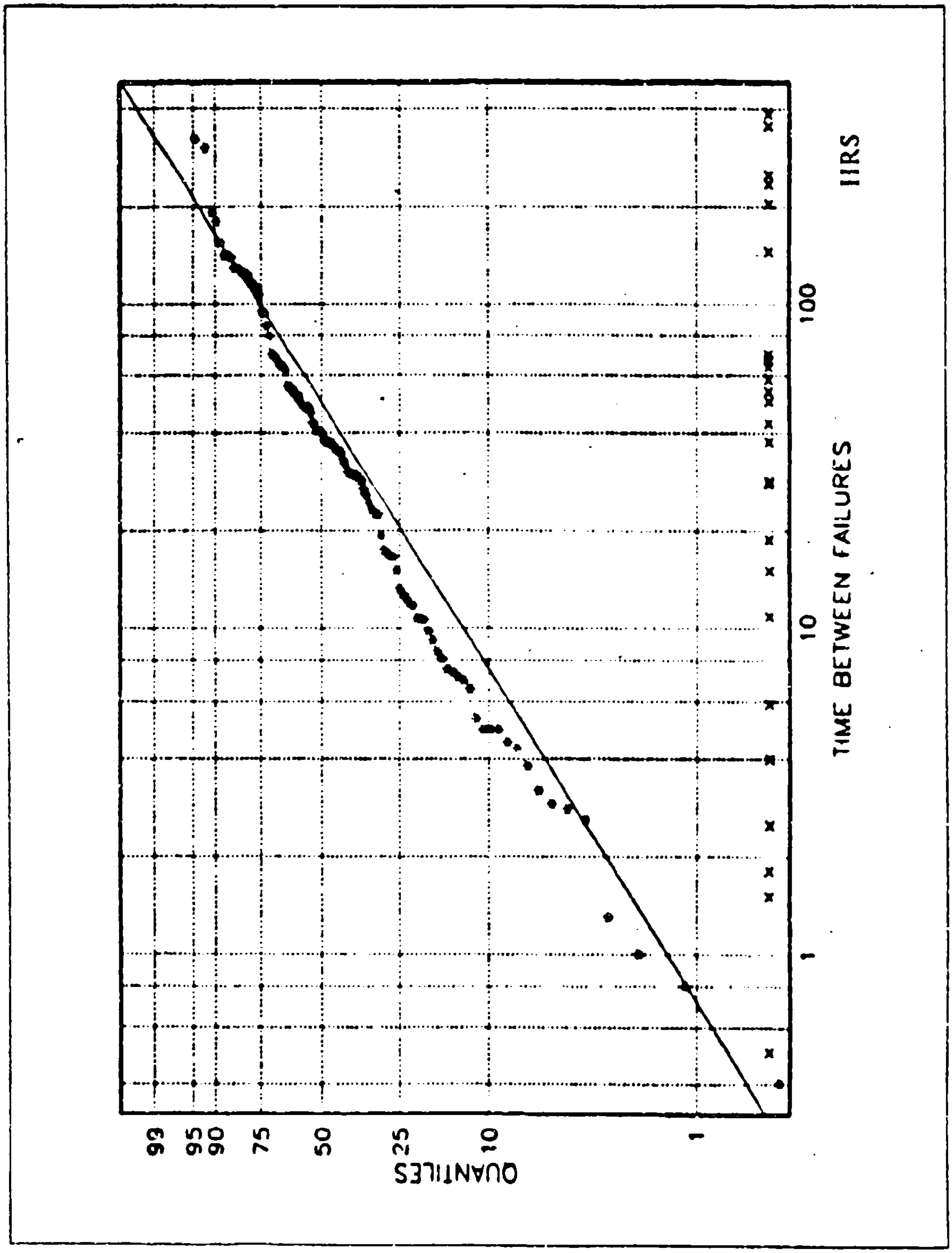

Figure 3.1 Exponential Quantile-Quantile Plot: WUC 734H100.

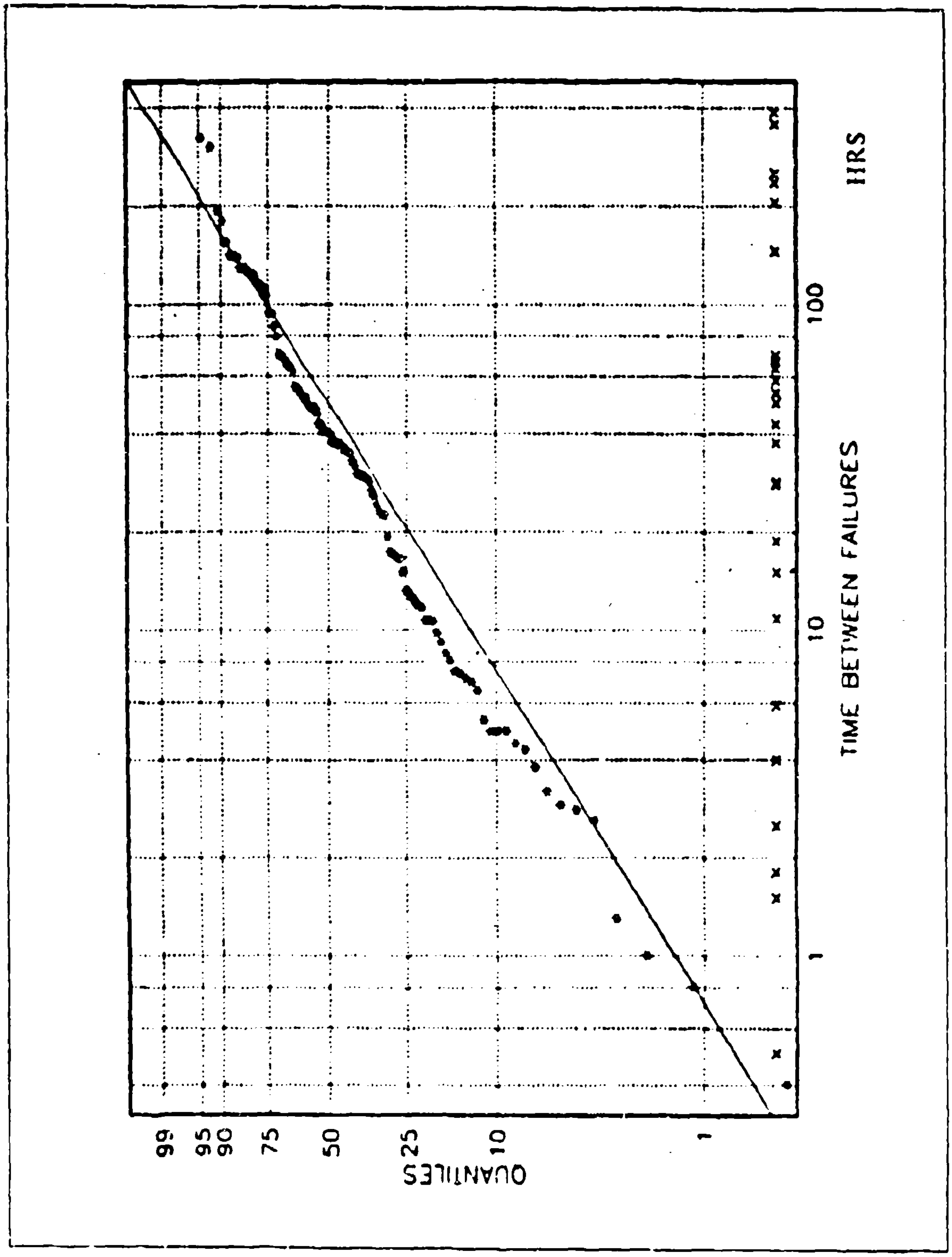

Figure 3.2 Exponential Quantile-Quantile Plot: WUC 56X2500.

identical situation displayed in each of the nine quantile-quantile plots. This would suggest that the distribution representing the time between component failures might be better modelled with a Weibull distribution with parameters, $\lambda > 0$ and $0 \leq \alpha < 1$.

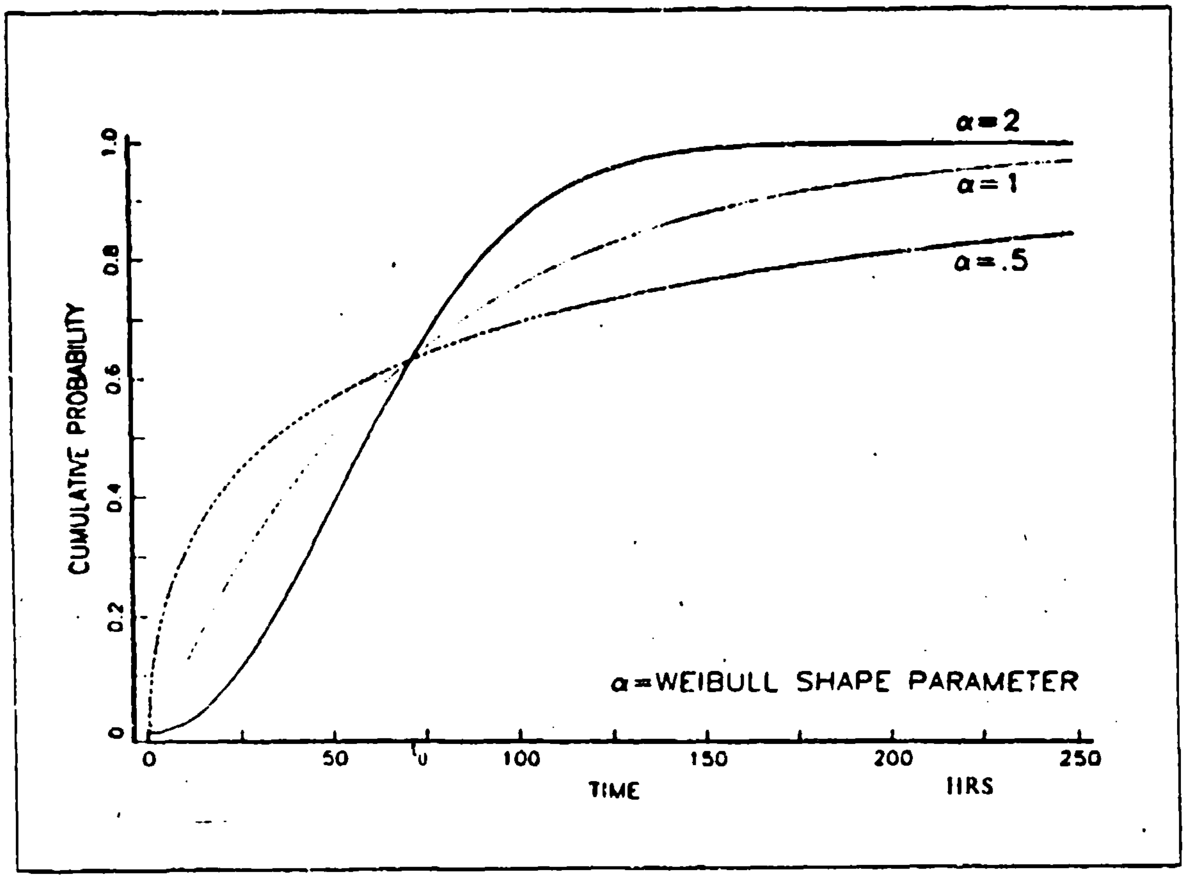

Figure 3.3 Weibull Cumulative Distribution Function.

C. WEIBULL MODEL

1. Properties

The next flight hour model that will be discussed is the Weibull distribution, which can be a viable alternative to the exponential distribution since it is capable of modelling many different failure rates. The major difference between the Weibull and Exponential models is that the former is usually not "memoryless", which indicates that the component's previous usage can be exploited in the determination of the expected failure time.

The two parameters associated with the Weibull distribution are the scale parameter (λ) and the shape parameter (α), with each shape parameter defining a specific failure rate distribution. As such, the Weibull family of distributions has the capability to model many different types of component wear. For example, α values greater than one describe a family of "WEAR OUT" distributions (increasing failure rate functions); an α value equal to one describe the "NO WEAR" exponential; and α values less than one describe a family of "WEAR IN" distributions (decreasing failure rate functions). Consequently, the Weibull distribution provides an additional modelling flexibility that should reduce the latter two areas of concern associated with the exponential distribution.

The Weibull cumulative distribution function (Figure 3.3) and the exponential quantile-quantile plots indicate that a Weibull model with scale parameter ($\lambda > 0$) and shape parameter ($\alpha < 1$) might provide appropriate descriptors of the failure rate distribution for each of the components studied. As mentioned above, a shape parameter less than one is indicative of a "WEAR IN" family of distributions. Distributions of this type are characterized by a hazard rate function, $r(t)$, which is decreasing in t, (i.e., the component's failure rate decreases as t increases). Since, the majority of components studied are electronic components (AVIONICS), this assumption of a decreasing failure rate is probably an improvement over the constant failure rate of the exponential model.

2. Maximum Likelihood Results

Maximum likelihood solutions can be obtained by setting the partials equal to zero, and solving for λ' and α' using nonlinear optimization techniques similar to the one described in Appendix G. The partial derivatives of the log-likelihood function with respect to the parameters λ and α are displayed below:[3]

$$\frac{\partial K}{\partial \lambda} = \sum_i \frac{\alpha\lambda^{\alpha-1}[(T_i + t_i)^{\alpha} - T_i^{\alpha}]S_{T_i}(t_i)}{1 - S_{T_i}(t_i)} - \sum_j \alpha\lambda^{\alpha-1}T_j^{\alpha}$$

$$\frac{\partial K}{\partial \alpha} = \sum_i \frac{\{[\lambda(T_i + t_i)]^{\alpha}\log[\lambda(T_i + t_i)] - (\lambda T_i)^{\alpha}\log(\lambda T_i)\}S_{T_i}(t_i)}{1 - S_{T_i}(t_i)} - \sum_j (\lambda T_j)^{\alpha}\log(\lambda T_j)$$

[3]The derivation of these equations is given in Appendix D.

where, $\alpha,\lambda > 0$; $T_i, t_i, T_j \geq 0$; $i = 1,2,\ldots,m$ (the set of uncensored observations - failures); and $j = m+1, m+2, \ldots, n$ (the set of censored observations).

The maximum likelihood estimates for λ' and α' were found using the FORTRAN program listed in Appendix G. The parameter estimates and 95 percent confidence limits for these estimates are displayed in Table 8.

TABLE 8
WEIBULL MAXIMUM LIKELIHOOD RESULTS

WUC	λ'	CL_λ	α'	CL_α
734H100	.0149	(.0084,.0216)	.8079	(.8014,.8146)
46X1600	.0044	(.0006,.0082)	.6664	(.6627,.6703)
4622100	.0066	(.0024,.0108)	.7708	(.7666,.7750)
56X2500	.0119	(.0063,.0176)	.8170	(.8114,.8227)
632Z100	.0067	(.0026,.0109)	.7970	(.7928,.8011)
74A1500	.0165	(.0084,.0246)	.7066	(.6985,.7147)
74A5M00	.0164	(.0104,.0224)	.8844	(.8784,.8904)
5772200	.0045	(.0009,.0082)	.7385	(.7349,.7422)
6918100	.0112	(.0058,.0167)	.8142	(.8087,.8196)

*** CL = 95 PERCENT CONFIDENCE LIMITS

3. Quantile-Quantile Plots

The parameter estimates listed in Table 8 were used to construct Kaplan-Meier quantile-quantile plots for each of the nine components studied. Two of these quantile-quantile plots are displayed in Figures 3.4 and 3.5, and the other seven are included in Appendix H. Both Figures graphically illustrate that the data is fit well by the Weibull distribution, with the worst case fit represented by Figure 3.4. This plot shows that the Weibull distribution has a tendency to underestimate the empirical quantiles below ten percent, but fits the data extremely well for quantiles greater than ten percent.

The results of the Weibull quantile-quantile plots are not conclusive, but offer credibility to the use of the Weibull model as an alternative method of modelling component failure rates when flight time is the only contributing factor.

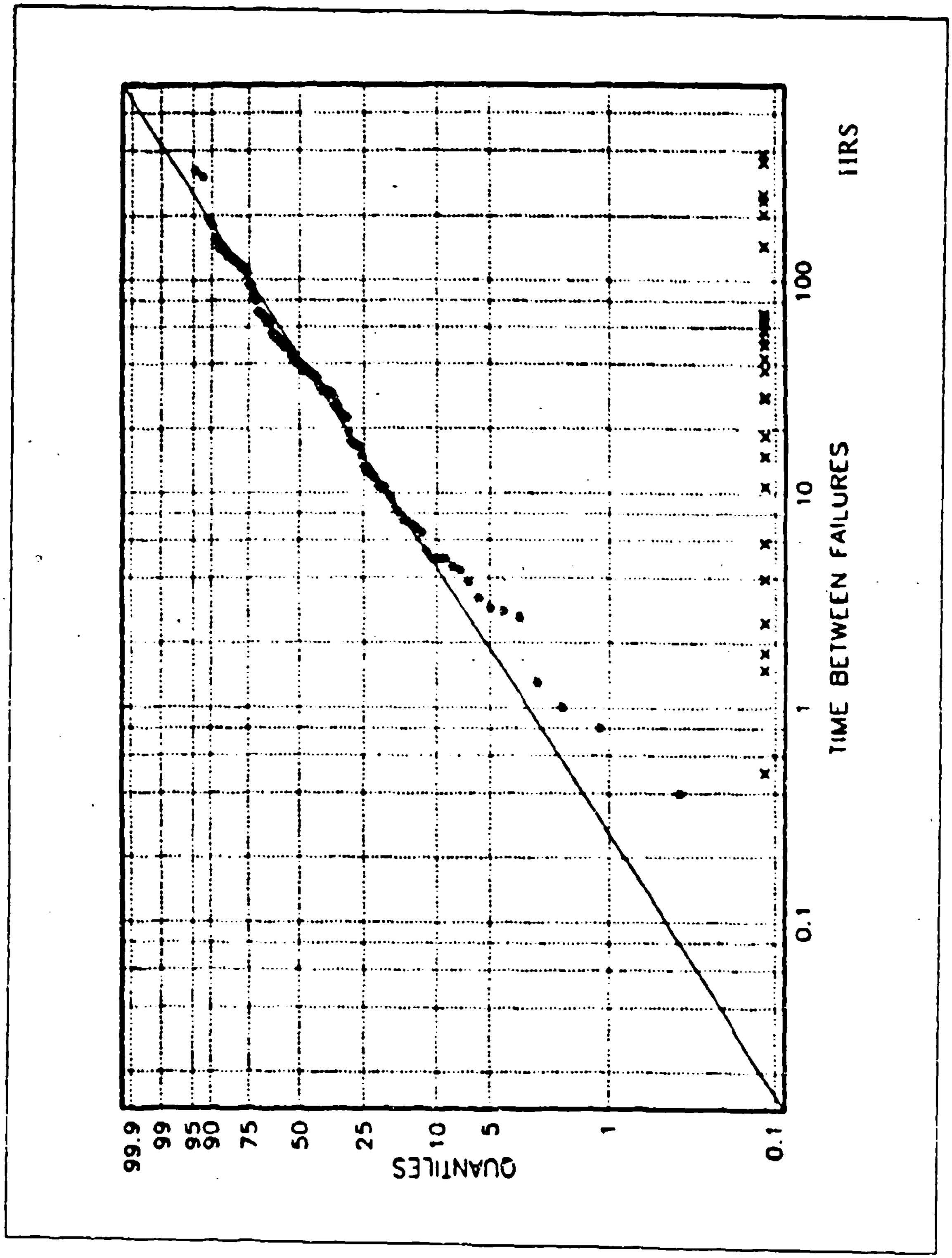

Figure 3.4 Weibull Quantile-Quantile Plot: WUC 73411100.

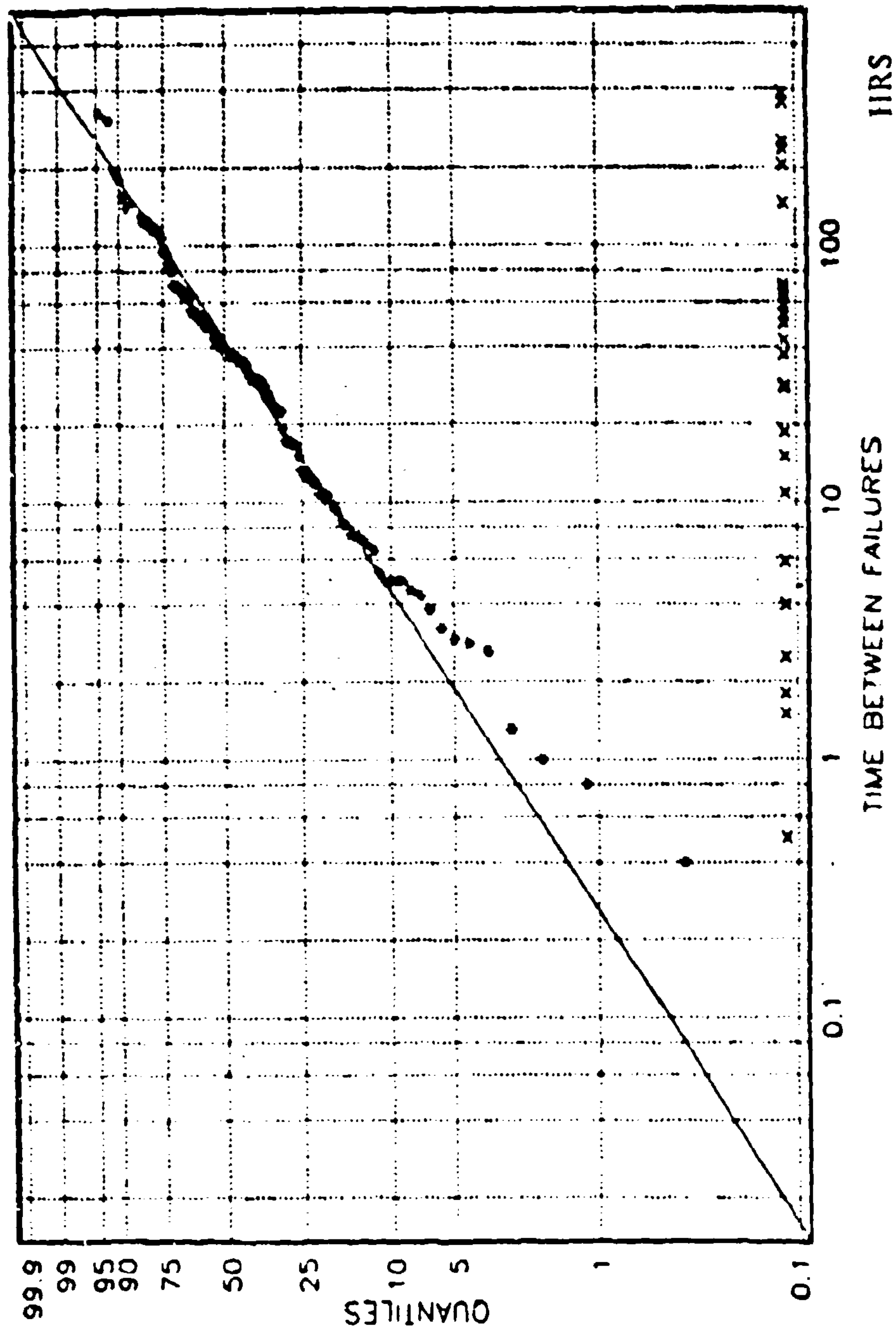

Figure 3.5 Weibull Quantile-Quantile Plot: WUC 56X2500.

IV. SORTIE DEPENDENT MODELS

A. OVERVIEW

The two models that will be discussed in this chapter differ from the models in Chapter III in that component failures can be affected by stresses incurred as a result of the sortie.[4] These stresses refer to events that occur during each flight, regardless of the sortie length, and would include events such as equipment initialization, takeoffs, landings, etc. The first sortie-dependent model to be discussed is the Geometric model, which determines a constant probability that a component fails on a given sortie.

The second sortie-dependent model that will be briefly looked at is the Mixed model described in reference 5. This model could also be classified as a flight hour model since it determines the probability of a component failure based on both sortie and flight hour contributing factors. It is incorporated in this section because it also utilizes the geometric distribution to determine the probability that the first failure occurs on the s^{th} sortie. The major difference between these two models is that the Mixed model's probability of failure for a given sortie is not constant. In fact, it is a function of a constant failure probability caused by sortie-related stress, and a probability of failure which is dependent on the sortie length.

B. GEOMETRIC MODEL

1. Properties

The basic premise of the Geometric model is that the component failure results due to sortie-related stress and is insensitive to differences in sortie length. This assumption can then be used to formulate the component's survival or failure for a given sortie as a Bernoulli trial, where, the component fails with probability p or survives with probability 1-p. The geometric distribution is used to determine the probability of observing the first failure on the s^{th} Bernoulli trial.

The geometric distribution is in many ways similar to the exponential distribution and might be considered the exponential's discrete analog. Like its continuous counterpart, the geometric distribution is "memoryless" and has a constant failure rate, which would indicate that the Geometric model also ignores the history of

[4]A sortie refers to one complete flight evolution, take-off to full-stop landing.

component usage. Any improvement offered by this model would be attributed to the effects of the sortie on the component's failure.

It should be apparent that a low variability in sortie length (i.e., the sortie length is approximately constant) would imply that the exponential distribution is equivalent to a geometric distribution with $p = 1 - \exp(-\lambda t)$; where t equals the average sortie length. In this case, both models would provide similar estimates of component failures. The flight data used to generate the parameter estimates was acquired from aircraft flying during Flex-deck operations, and should have greater variability than would be expected from aircraft flying under normal cyclic operations.[5]

2. **Maximum Likelihood Results**

The maximum likelihood expression for p', the estimate of the constant sortie failure probability is given in equation 4.1:[6]

$$p' = \frac{m}{\sum_i s_i} \qquad s_i = 1,2,\ldots \quad i = 1,2,\ldots,n \qquad \text{(eqn 4.1)}$$

where, m represents the number of uncensored observations (sorties with component failures), and the summation of the s_i's for $i = 1,2,\ldots,n$ represents the total number of sorties flown. Thus, the maximum likelihood estimator p' equals the number of flights experiencing a component failure divided by the total number of sorties flown. This is the same maximum likelihood estimate which would have resulted without censoring. The maximum likelihood estimates p' for p and the associated 95 percent confidence intervals are listed in Table 9.

C. MIXED MODEL

1. **Properties**

This section will provide a brief description of the Mixed model proposed by Levy [Ref. 5]. This particular model is structured such that the sortie and the sortie length (flight hours) can combine to cause a component failure. It is obvious that factors contributing to one component's failure may or may not be detrimental to another component's operating life. Most likely, "the life of electronic components is probably affected more by the number of power surges to which they are subjected

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[5]During Flex-deck operations, aircraft sortie length is not constrained to a one to two hour launch and recovery cycle.

[6]The derivation of this equation is given in Appendix D.

than operating hours; landing gear life should be influenced primarily by takeoffs and landings; and engine life should be affected by rotational speed changes, which are accompanied by heat transients and pressure changes" [Ref. 8: p. 2]. The Mixed model appears to have an increased flexibilty to model some of these sortie and flight hour factors.

As in the previous section, a constant probability of failure is employed to model the effects of sortie induced stress. In addition, the Mixed model incorporates a failure probability for those components whose lifetime is affected by the flight hours flown. Combining the constant sortie failure probability with the flight hour probability results in the expression for the probability of failure p_i for the i^{th} sortie, which is expressed as the complement of a component surviving the stresses of the sortie and the flight hours flown:

$$p_i = 1-\exp(-\lambda t_i)(1-p_0) \qquad 0 \le p_0 \le 1,\ \lambda > 0,\ t_i \ge 0 \qquad \text{(eqn 4.2)}$$

where, the flight hour failure distribution is exponential and p_0 equals the constant sortie failure probability.

2. **Maximum Likelihood Results**

The partial derivative of the log-likelihood function with respect to the parameters λ and p_0 are listed below:[7]

$$\frac{\partial K}{\partial \lambda} = \sum_i \frac{(1-p_0)t_i \exp(-\lambda t_i)}{1-\exp(-\lambda t_i)(1-p_0)} - T_i - \sum_j T_j \qquad \text{(eqn 4.3)}$$

$$\frac{\partial K}{\partial p_0} = \sum_i \frac{\exp(-\lambda t_i)}{1-\exp(-\lambda t_i)(1-p_0)} - \frac{s_i - 1}{1-p_0} - \sum_j \frac{s_j}{1-p_0} \qquad \text{(eqn 4.4)}$$

where, $0 \le p_0 \le 1$; $\lambda > 0$; $T_i, t_i, T_j \ge 0$; $s_i = 1,2,\ldots$; $i = 1,2,\ldots,m$ (the set of uncensored observations - failures); and $j = m+1, m+2, \ldots, n$ (the set of censored observations).

Maximum likelihood solutions for λ' and p_0' were obtained using the FORTRAN program listed in Appendix G. For each of the nine components studied, the log-likelihood function was maximized when λ' equaled zero, and $p_{0'}$ equaled p', the maximum likelihood estimate for the constant failure rate p of the Geometric

[7]The derivation of these equations is given in Appendix D.

model. The results for λ' are included in Table 9. The reader should note that the values for p' and p_0' are identical, and the failure for λ is zero for every component. Since both models provide the same maximum likelihood results, they will be jointly referred to as the Geometric model when compared with the other two models.

TABLE 9

MAXIMUM LIKELIHOOD RESULTS
SORTIE DEPENDENT MODELS

WUC	p'**	CL_{p_o}	λ'*	CL_{λ}
734H100	.0301	(.0188,.0414)	.0000	(.0000,.0055)
46X1600	.0097	(.0038,.0156)	.0000	(.0000,.0027)
4622100	.0140	(.0069,.0211)	.0000	(.0000,.0034)
56X2500	.0244	(.0147,.0341)	.0000	(.0000,.0047)
632Z100	.0140	(.0069,.0211)	.0000	(.0000,.0033)
74A1500	.0307	(.0191,.0423)	.0000	(.0000,.0056)
74A5M00	.0338	(.0228,.0448)	.0000	(.0000,.0053)
5772200	.0097	(.0037,.0157)	.0000	(.0000,.0028)
6918100	.0232	(.0137,.0327)	.0000	(.0000,.0045)

CL = 95 PERCENT CONFIDENCE LIMITS

* MIXED MODEL ONLY

** $p_o' = p'$

V. MODEL COMPARISONS

A. PURPOSE

The purpose of this chapter is to compare the Exponential model with the Weibull and Geometric models to determine if the distribution currently used provides the most accurate description of the failure process. As a method of comparison, the Chi-square goodness-of-fit test was not considered for two reasons. First, the failure rate information provided by the censored observations would have been lost because of the impossibility of determining the exact time of component failure. Second, it was considered more important to examine each model's capability to predict the number of component failures resulting from a specified number of sorties and flight hours flown. The next section describes the methods of estimating failures for each model. These estimates are then compared with the actual number of failures to determine each model's goodness-of-fit.

B. COMPONENT FAILURE ESTIMATION

1. Geometric Model

The determination of the expected number of component failures associated with the Geometric model is straightforward. The parameter p estimated using the Geometric model defines the probability that a component would fail during a given sortie. Furthermore, if it is assumed that the outcome of a sortie is independent of the outcome of the other N-1 sorties, and p is constant for each sortie, then the probability law defining the number of component failures for the deployment is the sum of N independent Bernoulli trials. This random sum has a Binomial distribution with an expected value equal to $N \times p$. Thus, using the Geometric model the expected number of component failures is calculated as the product of the number of sorties flown (N) and the probability that the component fails during the sortie.

2. Exponential Model

To compute the expected number of component failures using the Exponential model, each failure can be thought of as an occurrence within a Renewal process; a stochastic process which counts the number of occurrences within an interval (0,t]. It is assumed that the times between successive occurrences are independently and identically distributed non-negative random variables. As such, it can be shown that the expected number of occurrences within an interval (0,t] is equal to:

$$M(t) = \sum_k F_k(t) \qquad k = 1,2,3,...$$

where, M(t) equals the expected number of occurrences (failures) within the interval (0,t], and $F_k(t)$ is the k-fold convolution of the cumulative distribution function. The derivative of M(t) with respect to t defines the renewal rate of the process as:

$$M'(t) = m(t) = \sum_k f_k(t) \qquad k = 1,2,3,...$$

Since the k-fold convolution of the exponential probability density function is known to be Gamma, m(t) is equal to the infinite sum of Gamma probability density functions with scale parameter λ and shape parameter k.

$$m(t) = \sum_k \frac{\lambda^k x^{k-1} e^{-\lambda x}}{\Gamma(k)} = \lambda e^{-\lambda x} \sum_k \frac{(\lambda x)^{k-1}}{\Gamma(k)} \qquad k = 1,2,3,...$$

In the equation above, $\sum [(\lambda x)^{k-1}/\Gamma(k)]$ for $k = 1,2,3,...$ is easily recognized as the Taylor's series expansion of $e^{\lambda x}$. Thus, the renewal rate for the counting process defined by exponential interoccurrence times is λ. The expected number of failures M(t) is found by integrating m(t) from 0 to t.

$$M(t) = \int_0^t \lambda \, ds = \lambda t$$

Therefore, the expected number of component failures is equal to the product of the failure rate λ and the time on test t. This particular Renewal process is referred to as a Poisson process.

3. Weibull Model

Estimating the number of expected failures associated with the Weibull model is not as easy as the other methods discussed above. The observation of component failures is still a Renewal process, but the interoccurrence times are distributed according to a more complex distribution function. Finding the k-fold convolution of either the cumulative distribution function or the probability density function is an extremely difficult if not impossible task. For processes defined by continuous

interevent times with finite means and variances, the Elementary Renewal Theorem states that M(t) is approximately Normal with mean $= t/\mu$ and variance $= t\sigma^2/\mu^3$ as t approaches infinity [Ref. 9: p. 290]. Unfortunately, this theorem does not help estimate failures when t is small. As an alternate method to construct M(t) when t is small, a simple computer simulation is used.

The inputs required for the simulation included the number of aircraft assigned to the carrier air wings, the Weibull shape and scale parameters, and an approximate flight hour distribution. This information was used to repeatedly simulate the air wing operations during the deployment.

The aircraft were chosen at random to fly a fictitious mission of some sortie length t. For the modelling set, there were 30 aircraft assigned. The model assumed that each aircraft had an equally likely (1/30 percent) probability of being chosen to fly a mission. Realistically, aircraft are not chosen at random to fly missions, and most likely those aircraft recently flown are better prepared for future flights. A weighting scheme could have been used to increase the probabilities of those aircraft flown recently, and decrease the probability for those aircraft that were undergoing simulated maintenance activities.

The duration of the mission was determined by a random variable generated from a Normal distribution with the mean and variance estimated by the Model data set (see Table 6). Figure 5.1 depicts a histogram of the sortie lengths for the Model set with an overlay of the appropriate Normal distribution. Between the values of 0 to 6 hours, the Normal distribution fits the histogram adequately enough to justify its use as the sortie length's parent distribution. Values of randomly-generated sortie lengths less than .5 hours or greater than 6 hours were truncated to .5 hours and 6 hours, respectively.

After the selection of aircraft, the conditional probability of failure was computed for a flight of sortie length t. The only information required for this computation was the component's Weibull shape and scale parameters, and the aircraft's flight history since the last component failure. The conditional probability of failure could then be calculated as described in Appendix D. The value for the conditional probability of failure was then compared against a Uniform (0,1) random variable, and if the uniformly-generated outcome was less than or equal to the conditional probability of failure, a component failure was simulated.

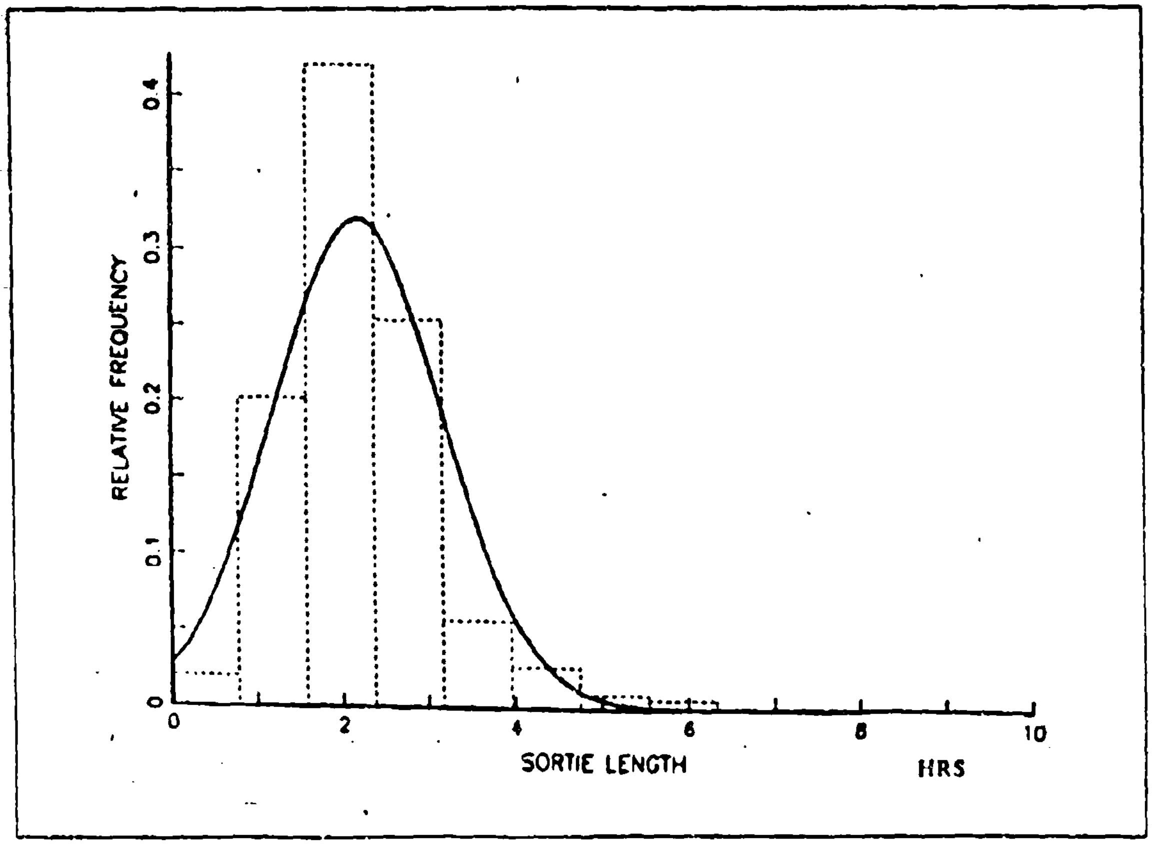

Figure 5.1 Sortie Length Histogram vs Normal Distribution.

The number of component failures was recorded at 500 hour intervals until the completion of the deployment. The deployment was simulated 100 times, and the average number of component failures was computed for each 500 hour interval. These averages were the approximations to the expected number of component failures $M(t)$ at 500,1000,1500,... hours etc.

C. COMPARISONS

1. Methodology

The predicted failures were compared with the actual failures as they occurred in each of the two deployments (USS KENNEDY and USS INDEPENDENCE). At this point, it is important to clarify the specific calculations of the expected failures. As described earlier, the accumulated flight time and total sorties flown were recorded with the observation of each failure. Therefore, the exponential and geometric expected failures could be computed using the formulas listed in this chapter. Unfortunately, the

expected number of Weibull component failures was not always defined at the accumulated flight time of the actual failure. To provide such an estimate, it was necessary to interpolate[8] between the Weibull expected failures associated with the simulated failure times above and below the actual time of component failure. The difference between the expected and actual failures was computed at the time of each component failure. These differences were then summed and divided by the total number of component failures to provide an estimate of the average prediction error. An average difference close to zero, with small standard deviation, indicates that the model is a good predictor of actual failures. A negative average shows that the model has a tendency to underestimate the number of component failures, and a positive average is indicative of overestimation. Tables 10 and 11 provide the average differences and corresponding standard deviations for the "model" and "validation" data sets respectively.

The model providing the best fit for a specific component was chosen based on the average difference between the predicted failures and the actual failures. Table 12 provides a summary listing of the optimal distributions for each component in both data sets. The reader is cautioned that the optimal was chosen based on an average difference closest to zero, and in some cases the expected number of failures was adequately described by other distributions as well as the one listed as optimal. If two or more models provided similar estimates, the one with the smallest standard deviation was chosen as optimal. The actual failures were plotted against the predicted failures to further illustrate a distribution's ability or disability to estimate failures. These graphs are included in Appendix I.

2. **Results**

Before proceeding with the comparisons, a caveat should be placed on the results. The ability of the models to predict the failures observed during the deployment of the "validation" data set is contingent on the accuracy of the model parameter estimates. For a valid comparison, it is necessary that both deployments come from the same population. That is to say, the flight operations were conducted under similar conditions. The reader should be aware that the "model" set aircraft and the "validation" set aircraft operated during different seasons and in different geographic locations. The parameters estimated from the "model" set may or may not provide the

[8]A quadratic interpolation of the form $aFT^2 + bFT + c = EF$ was used. Where, FT equaled accumulated flight time of failure, EF equaled expected failures, and a,b,c were parameters estimated using the known Weibull times and expected failures.

TABLE 10

MODEL SET AVERAGE DIFFERENCES

	Exponential	Geometric	Weibull
WUC:734H100			
Average:	-0.31	1.51	3.20
St.Dev.:	2.58	3.53	2.89
WUC:46X1600			
Average:	0.94	0.50	3.65
St.Dev.:	1.91	1.27	1.93
WUC:4622100			
Average:	-2.87	-1.91	0.56
St.Dev.:	2.54	1.97	2.52
WUC:56X2500			
Average:	-1.37	0.17	1.43
St.Dev.:	1.94	1.18	1.76
WUC:632Z100			
Average:	-0.04	0.08	2.74
St.Dev.:	1.85	2.12	2.29
WUC:74A1500			
Average:	-1.75	-0.03	5.78
St.Dev.:	4.77	3.92	5.69
WUC:74A5M00			
Average:	-0.53	1.05	0.09
St.Dev.:	3.81	4.45	4.14
WUC:5772200			
Average:	-1.49	-1.84	1.95
St.Dev.:	2.13	1.29	2.44
WUC:6918100			
Average:	-4.36	-2.97	-1.75
St.Dev.:	3.23	2.53	3.03

true estimate of the population parameters required to accurately predict failures within the "validation" data set.

Within the "model" set, all of the models predict failures adequately. This is not unexpected because the parameters defining the distribution were estimated using the flight data from the "model" set. Table 11 shows that this is not true for the

TABLE 11

VALIDATION SET AVERAGE DIFFERENCES

	Exponential	Geometric	Weibull
WUC:734H100			
Average:	20.76	16.44	26.22
St.Dev.:	11.01	8.85	12.01
WUC:46X1600			
Average:	-5.31	-8.04	-0.37
St.Dev.:	4.04	5.32	4.17
WUC:4622100			
Average:	16.20	14.57	20.90
St.Dev.:	15.33	13.95	16.86
WUC:56X2500			
Average:	-4.23	-7.41	0.48
St.Dev.:	4.01	5.15	3.49
WUC:632Z100			
Average:	-5.88	-8.43	-1.65
St.Dev.:	3.14	4.45	2.42
WUC:74A1500			
Average:	5.82	2.00	17.12
St.Dev.:	6.10	4.26	8.06
WUC:74A5M00			
Average:	8.67	2.83	10.39
St.Dev.:	3.41	3.75	3.74
WUC:5772200			
Average:	-4.61	-6.33	-0.07
St.Dev.:	1.93	2.40	2.02
WUC:6918100			
Average:	-2.01	-5.81	2.57
St.Dev.:	7.12	8.48	7.27

deployment defined by the "validation" data set. In most cases, a single optimal model can be identified.

Table 10 illustrates an extremely minor difference between the exponential and geometric estimates. In fact, it could be argued the geometric and the exponential were interchangeable as predictors of failures for the "model" set deployment. For the

TABLE 12
OPTIMAL DISTRIBUTIONS

WUC	MODEL SET	VALIDATION SET
734H100	Exponential	Geometric
46X1600	Geometric	Weibull
4622100	Weibull	Geometric
56X2500	Geometric	Weibull
632Z100	Exponential	Weibull
74A1500	Geometric	Geometric
74A5M00	Geometric	Geometric
5772200	Geometric	Weibull
6918100	Weibull	Exponential

Inertial Measuring Unit (WUC:734H100), the optimal distribution is listed as exponential for the "model" set and geometric for the "validation" set, but it could have been geometric in both cases. During the deployment represented by the "validation" data set the average sortie length increased by approximately 8.5 percent. As mentioned earlier, the exponential distribution is insensitive to changes in the average sortie length and would provide an estimate based on flight hours only. On the other hand, if the geometric distribution was correct the longer sortie lengths would imply that a specific flight hour total was accomplished with fewer sorties flown. It would provide a smaller estimate than would the Exponential model. For Work Unit Code 734H100, the Exponential model was unable to account for the longer average sortie length (in the "validation" set) and overestimated the actual number of failures.

The Geometric model appears to provide the best fit for components that are not necessarily operated continuously. Examples of components of this type would include the AWG-9 Radar Transmitter (WUC:74A1500) and Detail Data Display (WUC:74A5M00). A possible exception to this rule would be the Inertial Measuring Unit (WUC:734H100). Despite its continuous use, the Inertial Measuring Unit's gyros and accelerometers are extremely sensitive to the movements and impacts associated

with carrier takeoffs and landings, and it is extremely likely that component failures are caused by sortie-related stress vice continuous flight operations.

For continuous use components within the "validation" data set, the Weibull distribution provided the best estimate of component failures. The exponential distribution was the optimal predictor for only one component (WUC:6918100), and in that case, the Weibull distribution was nearly as accurate. From a readiness standpoint, the Weibull model might be considered a better choice because of its tendency to overestimate the actual number of failures. Amazingly, the exponential distribution predicted failures best for less than 10 percent of the components within the "validation" data set. In support of the current model, it should be noted that the exponential's estimate of failures was more than adequate for the "model" set.

As mentioned previously, the Weibull distribution with shape parameter equal to one is exponential. Figure 5.2 demonstrates the capability of the Weibull simulation to provide estimates similar to those generated for the Exponential model. This figure shows the Weibull and exponential predictions for the Detail Data Display (WUC:74A5M00) with estimated Weibull shape parameter equal to .91.

It should not be surprising that in some cases, none of the models will be adequate estimators of the actual failures. This type of phenomenon was observed for the Fuel Tank Release Mechanism (WUC:4622100). For two successive deployments, total failures went from 49 to 6 for a similar number of flight hours and sorties flown. Based on the available data, it is hard to determine if 49 was too high or 6 was too low. This type of outcome emphasizes the difficulty in predicting the actual failures for an upcoming deployment based on parameters estimated from one deployment rather than the entire historical data base.

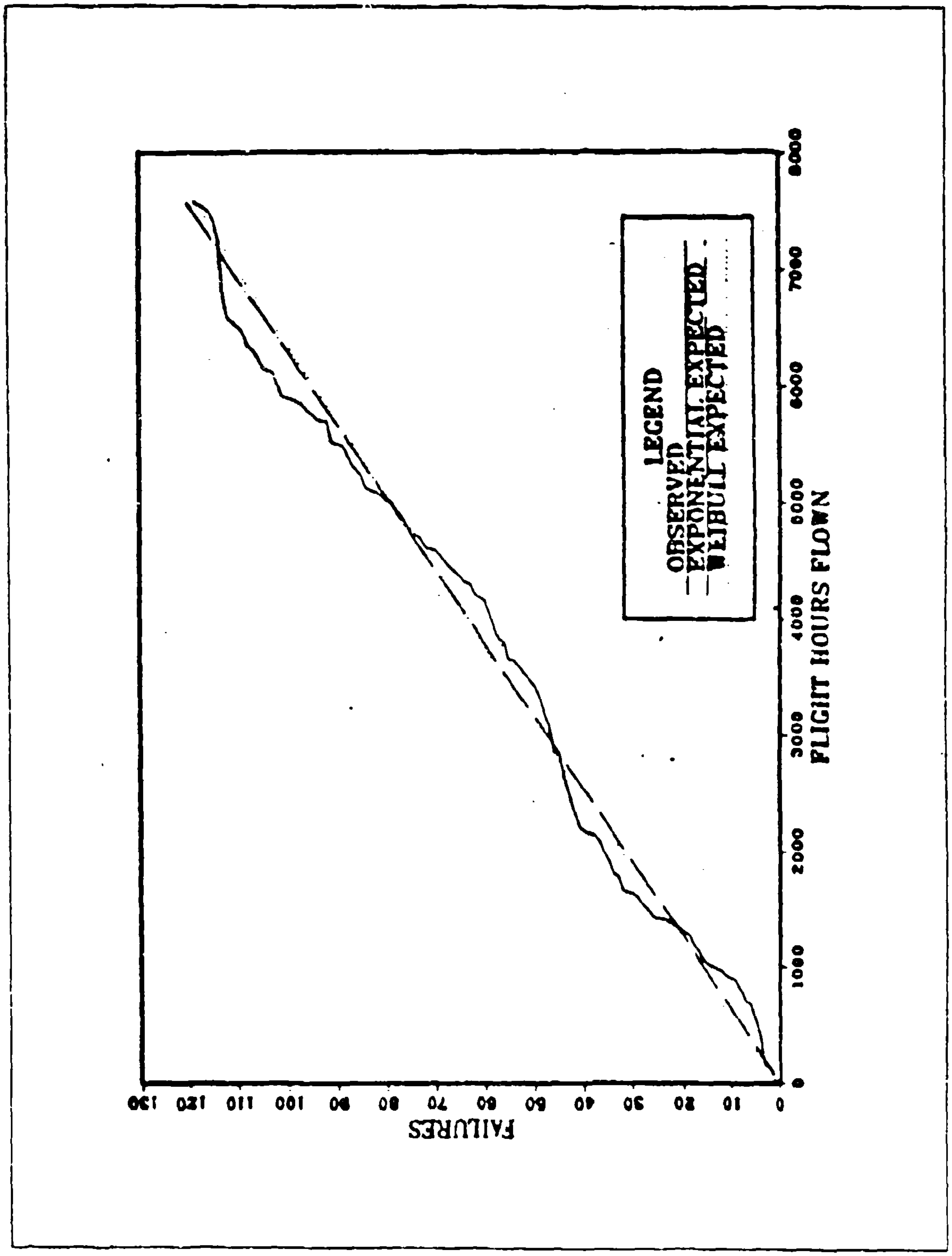

Figure 5.2 Flight Hour Models: Model Set WUC:74A5M00.

VI. SUMMARY

A. ADVANTAGES AND DISADVANTAGES

This section provides a brief summary of the primary advantages and disadvantages associated with each of the four models discussed in Chapters III and IV. The major advantage of the Exponential and Geometric models is that the number of expected failures is easily calculated using closed form expressions. Both models express failures as a linear function of only one factor, flight hours (Exponential) or sorties flown (Geometric). There is a tendency then to concentrate on aggregate flight totals while ignoring the specific flight history of each aircraft. As a result, neither model is able to adjust its failure estimates to compensate for differences in flight programs.

The Mixed model, on the other hand, attempts to account for failures resulting from sortie stress and/or continuous use. Unlike the previous models, the expected number of component failures is a function of a specific flight program. The probability of failure associated with the Mixed model is a function of the sortie duration. As such, this model should be more sensitive to differences in flight programs and would provide a more accurate estimate of failures than the Exponential and Geometric models. Use of the Exponential distribution to model continuous-use components has weakened the forecasting capabilities of the Mixed model. Lack of consideration for flight hours flown since the last component failure oversimplifies the problem. As stated previously, the small variability in sortie length combined with the "memoryless" property of the Exponential distribution would imply that the probability of flight hour induced failure is approximately constant for all sorties flown. Since the stress related failure probability is also constant, this model would not necessarily predict any better than the Geometric model. This type of phenomenon was observed in this study.

The Weibull's strongest feature is its ability to model failure rates for components which have increasing, decreasing, or constant failure rates. Since it does not possess the memoryless property of the Exponential/Geometric models, the expected number of failures is dependent upon previous flight histories. Like the Mixed model, this should provide an improved estimate of component failures. One disadvantage is a lack of

simple mathematical formulas with which to calculate the expected failures. However, in today's world of computers this should not be a major problem.

B. CONCLUSIONS AND RECOMMENDATIONS

It is extremely difficult to estimate the number of component failures which occur during a deployment based solely on the data provided by another deployment. Such an approach would not account for differences in flight and maintenance personnel, weather conditions, geographical location, mission type, aircraft's maintenance history, etc. For example, the number of actual failures observed for the Fuel Tank Release Mechanism went from forty-nine to six for a similar number of flight hours and sorties flown. To minimize the effects caused by such variations in deployments it is recommended that these models be evaluated using an enlarged data base.

Surprisingly, in this study, the model currently used by the Navy (Exponential) did not provide the best estimate of component failures. The Geometric provided a better fit for components which were not operated continuously, and the Weibull performed better when the components were operated continuously. In fact, overall, the Exponential was the least effective model for these nine components.

The results obtained using the Weibull simulation were encouraging. The model produced satisfactory estimates with a minimal number of assumptions, data input, and software coding. While the results obtained using this model were not overwhelming, an improvement in prediction capability was observed.

Two possible directions for improving the current method of estimating demand (i.e., component failures) are the construction of a universal model and the use of high resolution simulations. The first alternative suggests the use of a model which can describe components with different failure rate functions, and incorporates a methodology utilizing sortie-induced stress and flight hours flown to compute an estimate of component failures. The model currently used does not have this capability.

A slight modification to the Mixed model described in Chapter IV might provide the modelling flexibility desired. A failure rate distribution, such as the Weibull, could be used to describe continuous-use failures instead of the Exponential. Such a change would ensure that the component's failure probability could be influenced by prior use. That is to say, the probability of failure for a given sortie would now be a function of

the sortie length and the aggregate flight time since the last component failure. The strengths of the Weibull distribution combined with the constant probability of failure (sortie) associated with the Mixed model should provide an estimate of demand which is more sensitive to variations within and between flight programs.

The price paid for this improvement in forecasting ability is an increase in the complexity of the expected demand computations. With the abundance of computers within the fleet this should not be a problem. Perhaps, ten to twenty years ago it was necessary to rely on simple models for prediction and the crude estimates they provided. This is not the case today; computer simulations can be used when simple mathematical formulas are not available. As described in Chapter V, a low resolution simulation was used to forecast the component failures associated with the Weibull renewal process. The advantage of such simulations is the ability to include other factors which may affect a component's failure rate. Examples of these factors would include number and type of landings, mission type, maintenance programs, weather conditions, etc. It is safe to say, that these more complicated, high resolution models could include more information about the factors which cause components to fail, and should ultimately result in better forecasts of demand.

APPENDIX A
TRANSACTION CODES

CODE	DESCRIPTION
00	Used to report an inventory gain.
02	Used to report a change in the readiness reportable status of an equipment.
03	Used to report an equipment loss.
11	a. On-equipment work not involving a removal or replacement of a defective component or item. b. On supproting engine documents not having a removal of a defective or suspected defective component or item when the engine is not specifically identified to a specific aircraft. c. Used at the O or I level maintenance activities when closing out a maintenance action.
12	On-equipment work, including engines, involving non-repairable components items documented as failed parts.
14	Removal of a nondefective component/item, excluding cannibalization, from an engine to be processed at the O-level maintenance activity.
15	Installation of a nondefective component/item, excluding cannibalization, on an engine to be processed at an O-level maintenance activity.
16	Removal of a nondefective component/item, excluding engine components/items and a cannibalization to be processed at an O-level maintenance activity.
17	Installation of a nondefective component/item excluding engine component/items and cannibalization.
18	a. The removal and replacement of nondefective engines and component/items to accomplish a cannibalization action. b. The removal and replacement of those consumeable components/items subject to a scheduled removal interval or items of supply significance. c. The removal and replacement of a nondefective engine component for cannibalization at the I-level only.
19	a. The removal and replacement of nondefective engine component to accomplish a cannibalization at the O-level only. b. The removal and replacement of those consumeable engine components subject to a scheduled removal interval or items of supply significance.
21	Will be used when a repairable component is removed, excluding engine components, for processing at an I or D level maintenance activity (This code is used when only the removal must be documented and a replacement is not required).
23	Removal and replacement of a defective or suspected defective component from an end item, excluding engines at the O-level. Additionally, this code is used for the removal and

replacement of a complete engine assembly for a defect, suspected defect, or scheduled maintenance requirement.

24 Will be used when a repairable engine component is removed for processing at an I or D level activity (This code is used when only the removal must be documented and a replacement is not required).

25 Removal and replacement of a defective or suspected defective repairable component from an engine.

30 Used to document components processed through the I-level maintenance activity for check, test, and service.

31 Work performed on a removed repairable component with no failed parts or awaiting parts documented in the Failed/Required Material blocks of the VIDS/MAF.

32 Work performed on a removed repairable component with failed parts, awaiting parts, or cannibalization actions documented in the Failed/Required Material blocks.

39 Close out for man hours or awaiting parts at the I-level maintenance activity.

41 Technical directive compliance with no part number change.

47 Technical directive compliance with a part number change.

72 Will be useed to report SCIR data by the reporting custodian when transient maintenance is performed by other than the reporting custodian.

APPENDIX B
ACTION TAKEN CODES

CODE	DESCRIPTION
1	BCM - maintenance activity not authorized to conduct repair
2	BCM - Lack of equipment, tools, or facilities
3	BCM - Lack of technical skills necessary to complete repair
4	BCM - Lack of parts
5	BCM - Fails check and test, and maintenance is allowed to conduct check and test only
6	BCM - Lack of technical data
7	BCM - Beyond the authorized repair depth
8	BCM - Administrative
9	BCM - Condemned, repair not feasible
A	Items of repairable Material or Weapon/Support System Discrepency Checked No Repair Required. This code is used for all discrepencies which are checked and found that either the reported deficiency cannot be duplicated, or is operating within allowable tolerances.
B	Repair or replacement of attaching units, seals, gaskets, etc., that are not integral parts of work unit coded items.
C	This code is used when a repairable item of material identified by a work unit code is repaired.
D	This code is used to closeout a VIDS/MAF when component repair is to be performed at another facility.
F	Failure of components/items undergoing test.
J	This code is used when an item is calibrated and found serviceable without need for adjustment.
K	This code is used when an item must be adjusted to meet calibration standards.
L	This code is used when a maintenance action must be stopped or delayed while awaiting parts which are not available locally, and a component goes into an awaiting parts status.
N	This code is used by an organizational activity when it becomes necessary to closeout a maintenance action during or at the end of a reporting period for any reason.
P	This code is entered when an item of material is removed and only the removal is to be accounted for.

Q This code is entered when an item is installed and only the installation action is to be accounted for.

R This code is entered when an item of material is removed due to suspected malfunction and the same or a like item is reinstalled.

S This code is entered when an item of material is removed to facilitate other maintenance and the same item is reinstalled.

T This code is used when an item of material is removed and replaced for a cannibalization action.

Y This code is used when the time expended in locating a discrepency is great enough to warrant separating the troubleshoot time from the repair time.

Z This code is used when actually treating corroded items, and includes cleaning, treatment, priming and painting.

APPENDIX C
WHEN DISCOVERED CODES

CODE	DESCRIPTION
A	Before flight - mission aborted - failure discovered by aircrew
B	Before flight - no abort - failure discovered by aircrew
C	Inflight - mission aborted
D	Inflight - no abort
E	After flight/between flight - failure discovered by aircrew
F	Pilot/NFO weekly inspection
G	Acceptance/transfer inspection
H	Between flights - failure discovered by ground crew
J	Daily inspection
K	Preflight, daily/preflight, postflight, or turnaround inspection
L	Special inspection
M	Calender odd/major/phase inspection
N	Calender even inspection
O	Administrative
P	Functional checkflight
Q	Conditional inspection
R	Quality assurance inspection
S	Oil analysis inspection
U	Modification/SDLM/Overhaul/Airline maintenance
V	Related maintenance action
W	In-shop repair/disassembly for maintenance
X	Test bench/engine test stand operation
Y	Receipt or withdrawal from supply

APPENDIX D
MAXIMUM LIKELIHOOD DERIVATIONS

1. MAXIMUM LIKELIHOOD ESTIMATORS

a. Properties

The method used in this thesis to estimate the model parameters is maximum likelihood. As such, it is necessary to review two important properties of this estimator. First, if a random sample of size n is taken of a random variable X whose probability mass function, p(x), depends on an unknown parameter, θ, then the probability law for the maximum likelihood estimator is asymptotically normal (as n gets large) with:

$$\text{MEAN} = \theta \text{ and VARIANCE} = (nK^2)^{-1} \qquad \text{(eqn D.1)}$$

where, $K^2 = E[((\partial/\partial\theta)\ln p(x))^2]$ for a discrete random variable X, and n equals the total number of observations. Thus, the maximum likelihood estimator is asymptotically unbiased; that is, the expected value of the estimator is equal to the unknown parameter θ. Furthermore, the variance of the estimator is the Cramer-Rao lower bound, and represents the smallest possible variance of an unbiased estimator. [Ref. 10: pp. 372,379]

The second property describes the probability law for maximum likelihood estimators when the X_i's for $i = 1,\ldots,n$ are not identically distributed. In the models discussed in this thesis, the probability mass function for each X_i was dependent on one or more other random variables. Nevertheless, the distribution of the maximum likelihood estimator(s) is still asymptotically normal with:

$$\text{MEAN} = \theta \text{ and VARIANCE} = (\sum_i K_i^2)^{-1} \qquad \text{(eqn D.2)}$$

where, $K_i^2 = E[((\partial/\partial\theta)\ln p(x))^2]$ is evaluated using the specific parameters defining the i^{th} observation. The difference in the variance in equation D.1 and the variance in equation D.2 is caused by probability mass functions for the X_i's which are not identically distributed. For a complete derivation of this result, the reader is referred to W. J. Heintzelman's 1975 paper on *Determining the Failure Rate When Failure Times Are Not Known Exactly* [Ref. 11].

b. Confidence Intervals

Confidence intervals for the unknown parameters can then be calculated using the maximum likelihood's probability law described in the previous section. These limits are defined as:

$$\text{CONFIDENCE LIMITS} = \theta' \pm (z_{1-\alpha/2} \times \sigma_{\theta}')$$

where, θ' is the maximum likelihood estimator for θ, σ_{θ}' is the standard deviation of the maximum likelihood estimator, and $z_{1-\alpha/2}$ is the $1-\alpha/2$ quantile of the standard normal distribution.

c. Formulation of Probability Mass Functions

The generalized format used to construct each model's probability mass function for X_i is:

$$p(x_i) = [P(\text{failure occurred})]^{1-x_i} \times [P(\text{right hand censored})]^{x_i}$$

The random variable, X_i, is an indicator variable equal to zero if the data entry contains a failure, and equal to one if the observation is right hand censored.[9] The following variables are used to derive the maximum likelihood estimate(s) in each of the models:

- n : Total number of records in the final data set
- m : Total number of records containing component failures
- n-m : Total number of records which were right hand censored

2. EXPONENTIAL MODEL

a. Properties

The probability density function f(t), survival function S(t), and hazard rate function r(t) for the Exponential distribution are defined below.

$$f(t) = \lambda e^{-\lambda t} \qquad \lambda > 0$$

$$S(t) = Pr(T \geq t) = e^{-\lambda t}$$

$$r(t) = \frac{f(t)}{S(t)} = \lambda$$

[9] Right hand censored observations identify those components that did not fail prior to the termination of the deployment.

b. Formulation of the Maximum Likelihood Equation

This section outlines the formulation of the maximum likelihood equation used to find λ', the maximum likelihood estimate for the failure rate parameter λ. The final data set contains information supporting two record types. The first record type identifies observations of components failing during some specified interval of time, T_i to $T_i + t_i$. The probability equation representative of this type of record is given in equation D.3.

$$P(T_i \leq t \leq T_i + t_i) = \exp(-\lambda T_i) \times (1 - \exp(-\lambda t_i)) \qquad \text{(eqn D.3)}$$

The second record type pertains to an observation of a component surviving past some time T_i. These censored observations contain no information regarding the actual time of component failure. Equation D.4 defines the probability relationship associated with this type of observation.

$$S(T_i) = \exp(-\lambda T_i) \qquad \text{(eqn D.4)}$$

The probability mass function, $p(x_i)$, for the Exponential model is obtained by combining equations D.3 and D.4. The random variable X_i equals zero when a failure is observed and one when the data is right hand censored (i.e., component survived until the end of the deployment).

$$p(x_i) = [\exp(-\lambda T_i)]^{x_i} \times [\exp(-\lambda T_i) \times (1 - \exp(-\lambda t_i)]^{(1-x_i)} \qquad \text{(eqn D.5)}$$

where,

- $x_i = \{0,1\}$, $\lambda > 0$, and T_i, $t_i \geq 0$
- T_i equals the aggregate flight time without component failure for the ith record
- t_i equals the flight duration of the sortie during which the failure was observed

The likelihood function for $p(x_i)$, $L(p(x_i))$ can be expressed as the product of the n probability mass functions. The maximum likelihood estimate for λ is the value of λ' which maximizes $L(p(x_i))$. Since the logarithmic function is a monotonically increasing function, the λ' which maximizes $L(p(x_i))$ is identical to the λ' which maximizes the log-likelihood function $K(p(x_i))$. The optimal value for λ' is found by

taking the derivative of the log-likelihood function with respect to λ, setting it equal to zero, and solving for λ.

Likelihood function:

$$L(p(x_i)) = \prod_i [\exp(-\lambda T_i)]^{x_i} \times [\exp(-\lambda T_i) \times (1-\exp(-\lambda t_i)]^{(1-x_i)} \qquad \text{(eqn D.6)}$$

$i = 1,2,3,\ldots,n$

Log-likelihood function:

$$K(p(x_i)) = \sum_i -\lambda T_i x_i - \lambda T_i(1-x_i) + (1-x_i)\log[1-\exp(-\lambda t_i)] \qquad \text{(eqn D.7)}$$

The partial derivative of the log-likelihood function with respect to λ is:

$$\frac{\partial K}{\partial \lambda} = \sum_i -T_i x_i - T_i(1-x_i) + \frac{[(1-x_i)t_i\exp(-\lambda t_i)]}{[1-\exp(-\lambda t_i)]} \qquad \text{(eqn D.8)}$$

Equation. D.9 is the result of setting Equation D.8 equal to zero and rearranging the summations to eliminate the x_i(s). The left hand side of Equation D.9 is summed over the set (i) of all records, censored and uncensored. But, the right hand side is only summed for the uncensored observations (j).

$$\sum_i T_i = \sum_j \frac{t_j\exp(-\lambda t_j)}{1-\exp(-\lambda t_j)} \qquad \begin{array}{l} T_i, t_j \geq 0,\ \lambda > 0,\ m < n \\ i = 1,2,\ldots,n \quad j = 1,2,\ldots,m \end{array} \qquad \text{(eqn D.9)}$$

c. **Maximum Likelihood Variance**

As stated in Section D.1, the maximum likelihood estimator for λ is approximately normally distributed with mean equal to λ and variance equal to $1/\sum K_i^2$ for $i = 1,\ldots,n$. In this model:

$$K_i^2 = \frac{(t_i\exp(-\lambda t_i) - T_i)^2}{1-\exp(-\lambda t_i)} \times [\exp(-\lambda T_i) \times \exp(-\lambda t_i)] + T_i^2\exp(-\lambda T_i)$$

where, K_i^2 is evaluated using the maximum likelihood estimate of λ (i.e., $\lambda = \lambda^*$). The standard deviation of the maximum likelihood estimator and associated confidence limits displayed in Table 7 were computed using the equation listed above.

3. WEIBULL MODEL

a. Properties

The probability density function $f(t)$, survival function $S(t)$, and hazard rate function $r(t)$ for the Weibull distribution are displayed below.

$$f(t) = \lambda^{\alpha}\alpha t^{\alpha-1}\exp[-(\lambda t)^{\alpha}] \qquad \lambda,\ \alpha > 0 \quad t \geq 0$$

$$S(t) = \exp[-(\lambda t)^{\alpha}]$$

$$r(t) = \lambda^{\alpha}\alpha t^{\alpha-1}$$

b. Formulation of the Maximum Likelihood Equation

This section describes the formulation of the equation used to find the maximum likelihood estimates of the Weibull scale (λ) and shape (α) parameters. The two different record types supporting this equation are identical to those identified in the preceding section. The key difference between the two models is that the Weibull distribution possesses the "memoryless" property for α equal to one only. As such, the conditional survival probability is usually not equal to the unconditional survival probability. The conditional survival probability for the Weibull distribution is defined as:[10]

$$S_{T_i}(t_i) = \frac{\exp\{-[\lambda(T_i + t_i)]^{\alpha}\}}{\exp[-(\lambda T_i)^{\alpha}]}$$

The conditional probability of failure during the interval T_i to $T_i + t_i$ can be expressed as $1-S_{T_i}(t_i)$, the complement of the conditional survival probability. The probability mass function for the Weibull model is then formed by combining the conditional probability of a component failure within the interval T_i to $T_i + t_i$ ($x_i = 0$) with the probability that a component survives time T_i ($x_i = 1$).

$$p(x_i) = [1-S_{T_i}(t_i)]^{1-x_i} \times \exp[-(\lambda T_i)^{\alpha}]^{x_i} \qquad \text{(eqn D.11)}$$

where,

- $x_i = \{0,1\}$, $\lambda > 0$, and $T_i, t_i \geq 0$
- T_i equals the aggregate flight time without component failure for the ith record

[10]The subscripted T_i is used to indicate the conditional survival probability associated with a component that has survived T_i hours of operation.

- t_i equals the flight duration of the sortie during which the failure was observed

The log-likelihood function, $K(p(x_i))$, is:

$$K(p(x_i)) = \sum_i (1-x_i)\{\log[1-S_{T_i}(t_i)]\} + x_i[-(\lambda T_i)^{\alpha}] \qquad \text{(eqn D.12)}$$

$$i = 1,2,3,\ldots,n$$

The partial derivatives of the log-likelihood function, $K(p(x_i))$, with respect to the parameters λ and α are displayed below. They have been simplified by removing the x_i(s) and summing over the appropriate records. The set $i = 1,2,3,\ldots,n$ represents all data records (censored and uncensored), and the set $j = m+1, m+2, \ldots, n$ includes only those records which are censored.

$$\frac{\partial K}{\partial \lambda} = \sum_i \frac{\alpha\lambda^{\alpha-1}[(T_i+t_i)^{\alpha} - T_i^{\alpha}]S_{T_i}(t_i)}{1-S_{T_i}(t_i)} - \sum_j \alpha\lambda^{\alpha-1}T_j^{\alpha}$$

$$\frac{\partial K}{\partial \alpha} = \sum_i \frac{\{[\lambda(T_i+t_i)]^{\alpha}\log[\lambda(T_i+t_i)] - (\lambda T_i)^{\alpha}\log(\lambda T_i)\}S_{T_i}(t_i)}{1-S_{T_i}(t_i)} - \sum_j (\lambda T_j)^{\alpha}\log(\lambda T_j)$$

c. **Maximum Likelihood Variance**

The following equations were used to compute the variance for the maximum likelihood estimators, λ' and α'. These variances were then used to construct the confidence limits displayed in Table 8.

K_i^2 with respect to λ':

$$K_i^2(\lambda') = (\alpha\lambda^{\alpha-1})^2 \times \{\frac{\{[(T_i+t_i)^{\alpha} - T_i^{\alpha}]S_{T_i}(t_i)\}^2}{1-S_{T_i}(t_i)} + T_i^{2\alpha}\exp[-(\lambda T_i)^{\alpha}]\}$$

K_i^2 with respect to α':

$$K_i^2(\alpha') = \frac{(\{[\lambda(T_i+t_i)]^{\alpha}\log[\lambda(T_i+t_i)] - (\lambda T_i)^{\alpha}\log(\lambda T_i)\}S_{T_i}(t_i))^2}{1-S_{T_i}(t_i)} - \beta$$

where, β equals $[(\lambda T_i)^{\alpha}\log(\lambda T_i)]^2\exp[-(\lambda T_i)^{\alpha}]$.

4. GEOMETRIC MODEL

a. Properties

The probability mass function, $Pr(F=s)$, and survival function, $Pr(F \geq s)$, for the Geometric distribution are given below.

$$Pr(F=s)=pq^{s-1} \qquad 0 \leq p \leq 1,\ q=1-p \text{ and } s=1,2,3,\ldots$$
$$Pr(F \geq s)=q^{s-1}$$

where, F is the random variable representing the observation of the first failure, and q is the probability of the component surviving the sortie. Equations D.13 and D.14 illustrate the Geometric distribution's "memoryless" and constant failure rate properties respectively.

$$Pr(F>a+b \mid F>a) = (q^{a+b})/q^{a} = q^{b} = Pr(F>b) \qquad \text{(eqn D.13)}$$

$$Pr(F=s)/Pr(F \geq s) = (pq^{s-1})/q^{s-1} = p \qquad \text{(eqn D.14)}$$

b. Formulation of the Maximum Likelihood Equation

This section formulates the equation used to find the maximum likelihood stimate of the constant failure probability (p) associated with any given sortie. For the Geometric model, the two event types which can occur include components failing during the s^{th} sortie, or components failing during some unknown sortie after the s^{th} sortie. The probability expression associated with the first type of event is given in equation D.15, and equation D.16 defines the relationship for those observations where the component survived at least s sorties.

$$Pr(F=s) = pq^{s-1} \qquad \text{(eqn D.15)}$$

$$Pr(F>s) = q^{s} \qquad \text{(eqn D.16)}$$

The probability mass function for this model is obtained by combining equations D.15 and D.16. Again, X is allowed to take the values zero and one only, with zero indicating a component failure on the s^{th} sortie and a one indicating a component failure after the s^{th} sortie.

$$p(x_i) = [p(1-p)^{s_i-1}]^{1-x_i} \times [(1-p)^{s_i}]^{x_i} \qquad \text{(eqn D.17)}$$

where,

- $0 \le p \le 1$, $q = 1-p$, $s_i = 1,2,3,\ldots$
- s_i equals the aggregate number of sorties flown inclusive of the sortie causing the failure when $x_i = 0$ and equals the aggregate number of sorties without a component failure when $x_i = 1$

The maximum likelihood estimate p′ of the unknown sortie failure rate will be found by taking the derivative of the log-likelihood function $[K(p(x_i))]$ with respect to p, setting the derivative equal to zero and solving for p.

$$K(p(x_i)) = \sum_i (1-x_i)\ln(p) + (s_i-1)(1-x_i)\ln(1-p) + s_i x_i \ln(1-p) \qquad \text{(eqn D.18)}$$

$i = 1,2,3,\ldots,n$

$$\frac{\partial K}{\partial p} = \sum_i \frac{(1-x_i)}{p} - \frac{[(s_i-1)(1-x_i)]}{(1-p)} - \frac{(s_i x_i)}{(1-p)}$$

The partial derivative listed above can be simplified by removing the indicator variables and adjusting the summations to produce the closed form expression for p′ given in equation D.19.

$$\sum_j \frac{1}{p} + \frac{1}{1-p} = \sum_i \frac{s_i}{1-p}$$

$$p' = \frac{m}{\sum_i s_i} \qquad i = 1,2,\ldots,n \qquad \text{(eqn D.19)}$$

where, $j = 1,2,3,\ldots,m$ is the set of uncensored records (i.e., component failures), and $i = 1,2,3,\ldots,n$ is the set of all records, censored and uncensored. Thus, the maximum likelihood estimator p′ equals the number of flights experiencing a component failure divided by the total number of sorties flown.

c. Maximum Likelihood Variance

The maximum likelihood estimator's variance was computed using the formulas described in Section D.1. The equation defining the Geometric model's expression for K_i^2 is given below:

$$K_i^2 = [(1-ps_i)^2(1-p)^{s_i-3}]/p + s_i^2(1-p)^{s_i-2} \qquad \text{(eqn D.20)}$$

where, K_i^2 is evaluated at $p = p'$. These variances were used to compute the confidence limits listed in Table 9.

5. MIXED MODEL

a. Properties

Combining a constant sortie failure probability, p_0, with a flight hour probability distribution results in an expression for the probability of failure, p_i, for the i^{th} sortie. This is expressed as the complement of a component surviving both the stresses of the sortie and the flight hours flown:

$$p_i = 1-\exp(-\lambda t_i)(1-p_0) \qquad 0 \le p_0 \le 1,\ \lambda > 0,\ t_i \ge 0 \qquad \text{(eqn D.21)}$$

where, the flight hour failure distribution is Exponential.

The model is then formulated in the same manner as the Geometric model described in the preceding section. The probability that the first failure occurs on the s^{th} sortie is expressed in equation D.22. Note that the $p_{i(s)}$ are not necessarily equal for $i = 1,\ldots,s-1$.

$$\Pr(F=s) = p_s \prod_i (1-p_i) \qquad i = 1,2,\ldots,s-1 \qquad \text{(eqn D.22)}$$

Equation D.23 results from using the expression for p_i given in equation D.21 to rewrite equation D.22.

$$\Pr(F=s) = [1-\exp(-\lambda t)(1-p_0)][(1-p_0)^{s-1}\exp(-\lambda T)] \qquad T_i,t_i \ge 0 \qquad \text{(eqn D.23)}$$

b. Formulation of the Maximum Likelihood Equation

This section describes the formulation of the equation used to find the maximum likelihood estimates for the failure rate parameter (λ) and the constant sortie probability of failure (p_0). The probability relationship supporting the observation of the first failure on the s^{th} sortie is given in equation D.23, and the expression for the observation of a failure at some unknown time after s^{th} sortie is listed below.

$$Pr(F > s) = \prod_i (1-p_i) = \exp(-\lambda T)(1-p_0)^s \qquad \text{(eqn D.24)}$$

The probability mass function $p(x_i)$ for this model results from combining equations D.23 and D.24. Notice that when λ equals zero the probability mass function for the Mixed model is identical to the probability mass function expressed for the Geometric model in equation D.17, and when p_0 equals zero the probability mass function is equivalent to the Exponential probability mass function given in equation D.5. The above statement implies that the Mixed model has the capability of modelling the Geometric or Exponential cases as the situation would require.

$$p(x_i) = \{[1-\exp(-\lambda t_i)(1-p_0)][(1-p_0)^{s_i-1}\exp(-\lambda T_i)]\}^{1-x_i} \times [(1-p_0)^{s_i}\exp(-\lambda T_i)]^{x_i}$$

where,

- $0 \le p_0 \le 1$, $\lambda > 0$; $s_i = 1,2,3,\ldots,$ and T_i, $t_i \ge 0$
- T_i equals the aggregate flight time without component failure for the ith record
- t_i equals the flight duration of the sortie during which the failure was observed
- s_i equals the aggregate number of sorties flown inclusive of the sortie causing the failure when $x_i = 0$ and equals the aggregate number of sorties without a component failure when $x_i = 1$.

The log-likelihood function $K(p(x_i))$, and the partial derivatives of the log-likelihood functions with respect to the parameters λ and p_0 are given below. The maximum likelihood estimates for λ and p_0 were obtained utilizing the Quasi-Newton method described in Appendix G. The log-likelihood function, $K(p(x_i))$, is:

$$K(p(x_i)) = \sum_i (1-x_i)\{\ln[1-\exp(-\lambda t_i)(1-p_0)] + [(s_i-1)\ln(1-p_0)] - \lambda T_i\} + x_i\{[s_i \ln(1-p_0)] - \lambda T_i\}$$
$$i = 1,2,3,\ldots,n$$

The partial derivatives of the log-likelihood function, $K(p(x_i))$, have been simplified by removing the x_i(s) and summing over the appropriate records. The set $i = 1,2,3,\ldots,n$ represents all data records (censored and uncensored), and the set $j = m+1, m+2,\ldots,n$ includes only those records which are censored.

$$\frac{\partial K}{\partial \lambda} = \sum_i \frac{(1-p_o)t_i \exp(-\lambda t_i)}{1-\exp(-\lambda t_i)(1-p_o)} - T_i - \sum_j T_j \qquad \text{(eqn A.25)}$$

$$\frac{\partial K}{\partial p_o} = \sum_i \frac{\exp(-\lambda t_i)}{1-\exp(-\lambda t_i)(1-p_o)} - \frac{s_i-1}{1-p_o} - \sum_j \frac{s_j}{1-p_o} \qquad \text{(eqn A.26)}$$

c. Maximum Likelihood Variance

The equation for the variance of the probability distribution for p_o' is identical to equation D.20 when λ equals zero. The equation for the K_i^2 for λ' is given below.

$$K_i^2(\lambda') = [\{(1-p_o)t_i]/p_o - T_i\}^2 \times p_o(1-p_o)^{s_i-1} + T_i^2(1-p_o)^{s_i}$$

where, K_i^2 is evaluated at $\lambda = \lambda' = 0$. These variances were used to compute the confidence limits for λ listed in Table 9.

APPENDIX E

SINGLE PARAMETER OPTIMIZER

```
*    COMPUTES MAXIMUM LIKELIHOOD VALUE FOR A SINGLE UNKNOWN
*    PARAMETER USING GOLDEN RATIO LINE SEARCH.

        IMPLICIT REAL*8 (A-H,O-Z)
        COMMON/RL/FTNF,FTF,PARAM,PSD
        COMMON/INT/CENSOR,NF,COUNT
        DIMENSION CENSOR(200),NF(200),FTNF(200),FTF(200),PARAM(2)
        INTEGER CENSOR,COUNT,TFLAG
        REAL*8 LB,LC

        DATA LB/.0001D0/ITER/1/K/1/STEP/.00001D0/ALPHA/.0001D0/

*    INITIALIZE
        COUNT=1
        PARAM(1)=.00001D0
        TFLAG=0

*    COMPUTATION OF SEARCH INTERVAL TAU. TAU IS APPROXIMATELY
*    .618 AND TAU1 IS APPROXIMATELY .382
        TAU=(5**.5-1)/2
        TAU1=1-TAU

*    INPUT DATA
5     READ (3,50,END=99) CENSOR(COUNT),NF(COUNT),FTNF(COUNT),FTF(COUNT)
        COUNT=COUNT+1
        GO TO 5
99    COUNT=COUNT-1

*    BOUNDING PHASE - SWANN'S METHOD
        CALL ML(LB,F1)
6     ALPHA=ALPHA+(2**K)*STEP
        CALL ML(ALPHA,F2)

*    IF NEW VALUE IS LESS THAN PREVIOUS VALUE TERMINATE BOUNDING
        IF (F2.LT.F1) GO TO 10
        F1=F2
        K=K+1
        GO TO 6
10    UB=ALPHA

*DEFINE INITIAL POINTS & ASSOCIATED FUNCTIONAL VALUES

        B1=TAU1*(UB-LB)+LB
        B2=TAU*(UB-LB)+LB
        CALL ML(B1,F1)
        CALL ML(B2,F2)

*    COMPARES THE FUNCTIONAL VALUES OF THE INTERNAL PTS. B1 & B2, AND
*    SELECTS THE VALUE WITH THE LARGEST FUNCTIONAL VALUE AS THE UPDATED
*    PARAMETER ESTIMATE

150   IF (F1.GT.F2) THEN
            PARAM(2)=B1
            UB=B2
            B2=B1
            F2=F1
            B1=TAU1*(UB-LB)+LB
            CALL ML(B1,F1)
            ITER=ITER+1
            GO TO 200
        END IF
```

```
          IF (F2.GE.F1) THEN
             PARAM(2)=B2
             LB=B1
             B1=B2
             F1=F2
             B2=TAU*(UB-LB)+LB
             CALL ML(B2,F2)
             ITER=ITER+1
             GO TO 200
          END IF

200      CALL TERM(F1,F2,TFLAG)
          PARAM(1)=PARAM(2)
          IF (TFLAG.EQ.1) GO TO 275
          GO TO 150

*     WRITE OUTPUT
275      WRITE (6,300)
          WRITE (6,310) PARAM(1)
          CALL SD
          WRITE (6,305) PSD
          UC=PARAM(1)+1.96*PSD
          LC=PARAM(1)-1.96*PSD
          WRITE (6,320) LC,UC

*     FORMAT
50       FORMAT (I1,1X,I4,5X,F6.1,1X,F3.1)
300      FORMAT ('OWUC:'/'OCONVERGENCE CRITERION ESTABLISHED')
305      FORMAT ('OSTANDARD DEVIATION IS        : ',F9.8)
310      FORMAT ('OMLE FOR UNKNOWN PARAMETER IS: ',F9.8)
320      FORMAT ('0.95 ASSYMPTOTIC CONFIDENCE LIMITS ARE :',F5.4,',',F5.4)
9999     STOP
          END

          SUBROUTINE TERM(F1,F2,TFLAG)

*     DETERMINES IF TERMINATION CONDITION HAS BEEN SATISFIED

          IMPLICIT REAL*8 (A-H,O-Z)
          COMMON/RL/FTNF,FTF,PARAM,PSD
          COMMON/INT/CENSOR,NF,COUNT
          DIMENSION CENSOR(200),NF(200),FTNF(200),FTF(200),PARAM(2)
          INTEGER TFLAG

          TEST=ABS(F1-F2)
          IF (TEST.LE..00003) TFLAG=1

          RETURN
          END

          SUBROUTINE ML(PT,FVAL)

*     COMPUTES LOG-LIKELIHOOD FUNCTIONAL VALUE

          IMPLICIT REAL*8 (A-H,O-Z)
          COMMON/RL/FTNF,FTF,PARAM,PSD
          COMMON/INT/CENSOR,NF,COUNT
          DIMENSION CENSOR(200),NF(200),FTNF(200),FTF(200),PARAM(2)
          INTEGER CENSOR,COUNT,TFLAG
          REAL*8 K1

          FVAL=0

*     INPUT FLIGHT DATA
          DO 5 I=1,COUNT

*     UNCENSORED RECORDS
```

```
      IF (CENSOR(I).EQ.0) THEN
         K1=EXP(-PT*FTF(I))
         FVAL=FVAL+LOG(1-K1)-PT*FTNF(I)
         GO TO 5
      END IF

*   CENSORED RECORDS
      FVAL=FVAL-PT*FTNF(I)
5    CONTINUE

      RETURN
      END


      SUBROUTINE SD

*   COMPUTES ASSYMPTOTIC STANDARD DEVIATION FOR THE MAXIMUM LIKELIHOOD
*   ESTIMATE

      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/RL/FTNF,FTF,PARAM,PSD
      COMMON/INT/CENSOR,NF,COUNT
      DIMENSION CENSOR(200),NF(200),FTNF(200),FTF(200),PARAM(2)
      INTEGER CENSOR,COUNT
      REAL*8 KISQ,K1,K2,K3

      SUMKI=0.

*   INPUT FLIGHT DATA
      DO 5 I=1,COUNT

*   UNCENSORED RECORDS
      IF (CENSOR(I).EQ.0) THEN
         K1=EXP(-PARAM(1)*FTF(I))
         K2=EXP(-PARAM(1)*FTNF(I))
         K3=((FTF(I)*K1/(1-K1))-FTNF(I))*((FTF(I)*K1/(1-K1))-FTNF(I))
         KISQ=K3*(K2*(1-K1))+(FTNF(I)*FTNF(I)*K2)
         GO TO 6
      END IF

*   CENSORED RECORDS
      KISQ=FTNF(I)*FTNF(I)*K2
6    SUMKI=SUMKI+KISQ
5    CONTINUE

      PSD=SQRT(1/SUMKI)

      RETURN
      END


      SUBROUTINE SD1

*   COMPUTES ASSYMPTOTIC STANDARD DEVIATION FOR THE MAXIMUM LIKELIHOOD
*   ESTIMATE FOR THE SORTIE DEPENDENT MODELS

      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/RL/FTNF,FTF,PARAM,PSD
      COMMON/INT/CENSOR,NF,COUNT
      DIMENSION CENSOR(200),NF(200),FTNF(200),FTF(200),PARAM(2)
      INTEGER CENSOR,COUNT
      REAL*8 L,LL1,LL2

*   INITIALIZE PARAMETER ESTIMATES
      P=.0232D0
      L=.0000D0
      SUMKI1=0.0D0
      SUMKI2=0.0D0
```

```
*     INPUT FLIGHT DATA
         DO 5 I=1,COUNT
         C1=(1-P*NF(I))
         C2=(1-P)**NF(I)
         C3=C2/(1-P)
         C4=C3/(1-P)
         C5=C4/(1-P)
         C6=((1-P)*FTF(I))/P-FTNF(I)
         SUMKI1=SUMKI1+(C1*C1*C5)/P+NF(I)*NF(I)*C4
         SUMKI2=SUMKI2+C6*C6*P*C3+FTNF(I)*FTNF(I)*C2
5       CONTINUE

*     COMPUTE STANDARD DEVIATIONS
         PSD1=SQRT(1/SUMKI1)
         PSD2=SQRT(1/SUMKI2)

*     COMPUTE CONFIDENCE LIMITS
         LL1=P-1.96*PSD1
         UL1=P+1.96*PSD1
         LL2=L-1.96*PSD2
         UL2=L+1.96*PSD2

*     OUTPUT
         WRITE (6,320) LL1,UL1
320     FORMAT ('0.95 ASSYMPTOTIC CONFIDENCE LIMITS FOR P: ',F5.4,',',F5.4
        C)
         WRITE (6,330) LL2,UL2
330     FORMAT ('0.95 ASSYMPTOTIC CONFIDENCE LIMITS FOR LAMBDA: ',F5.4,','
        C,F5.4)

         RETURN
         END
```

APPENDIX F
QUANTILE-QUANTILE PLOTS

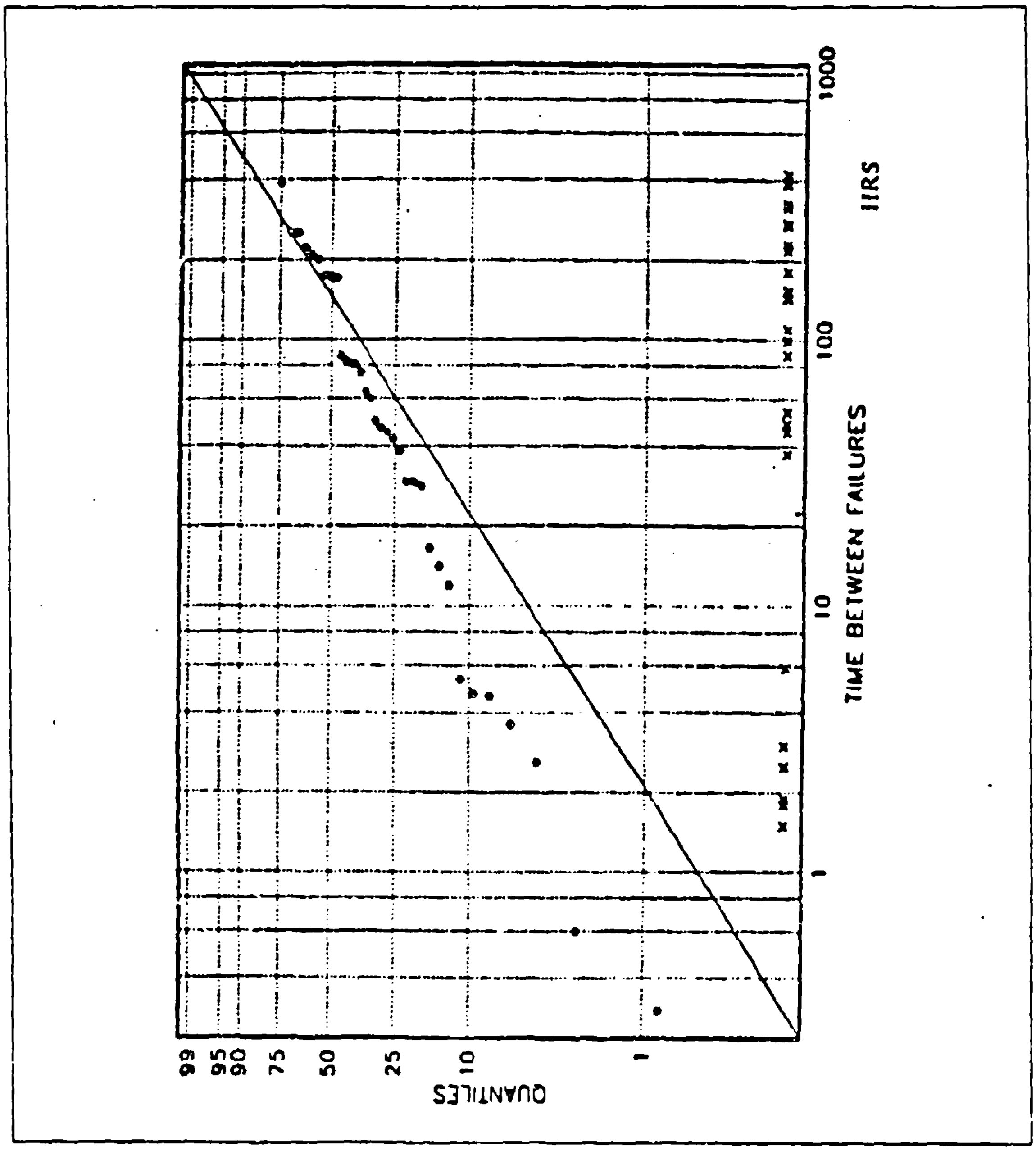

Figure F.1 Exponential Quantile-Quantile Plot: WUC 46X1600.

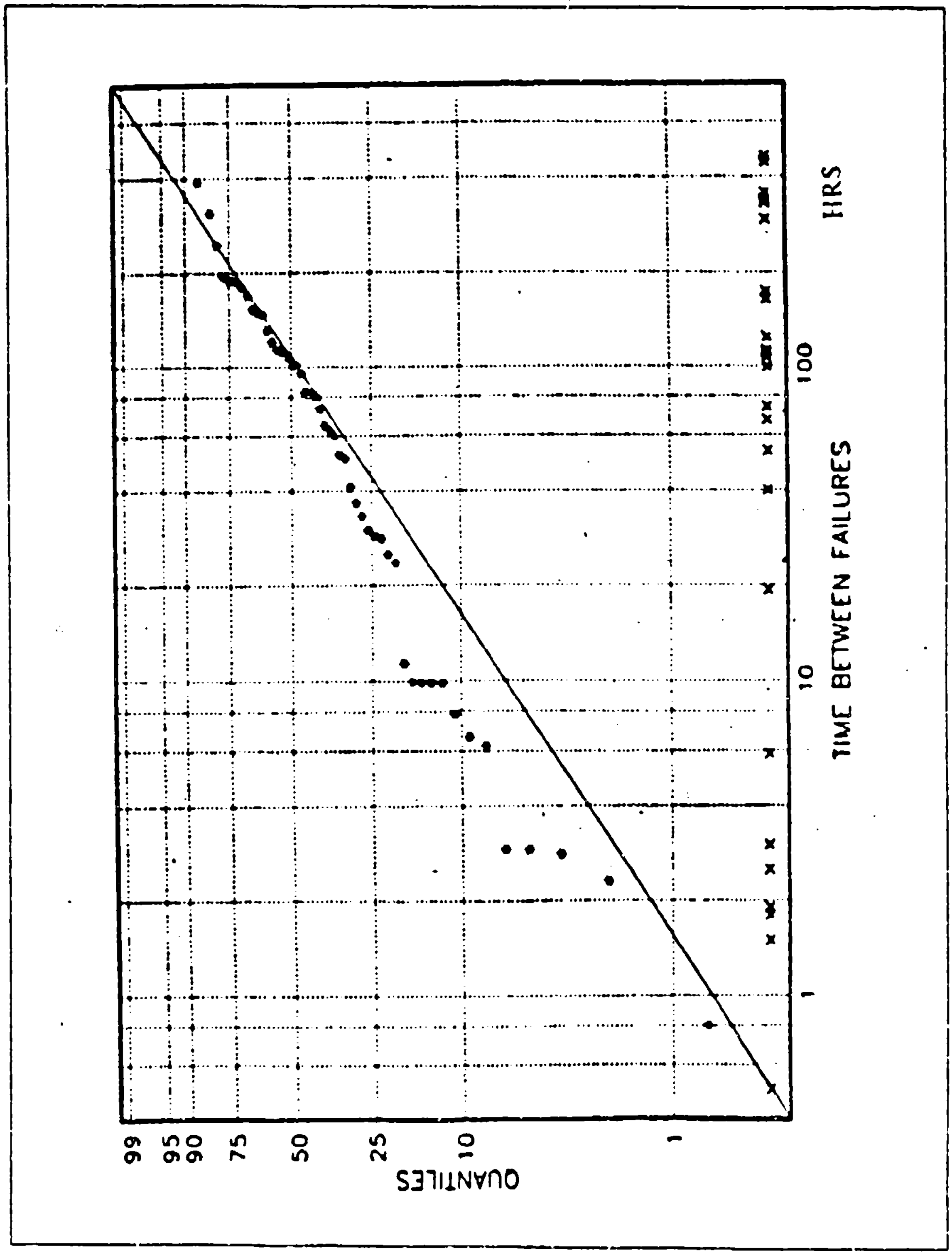

Figure F.2 Exponential Quantile-Quantile Plot: WUC 4622100.

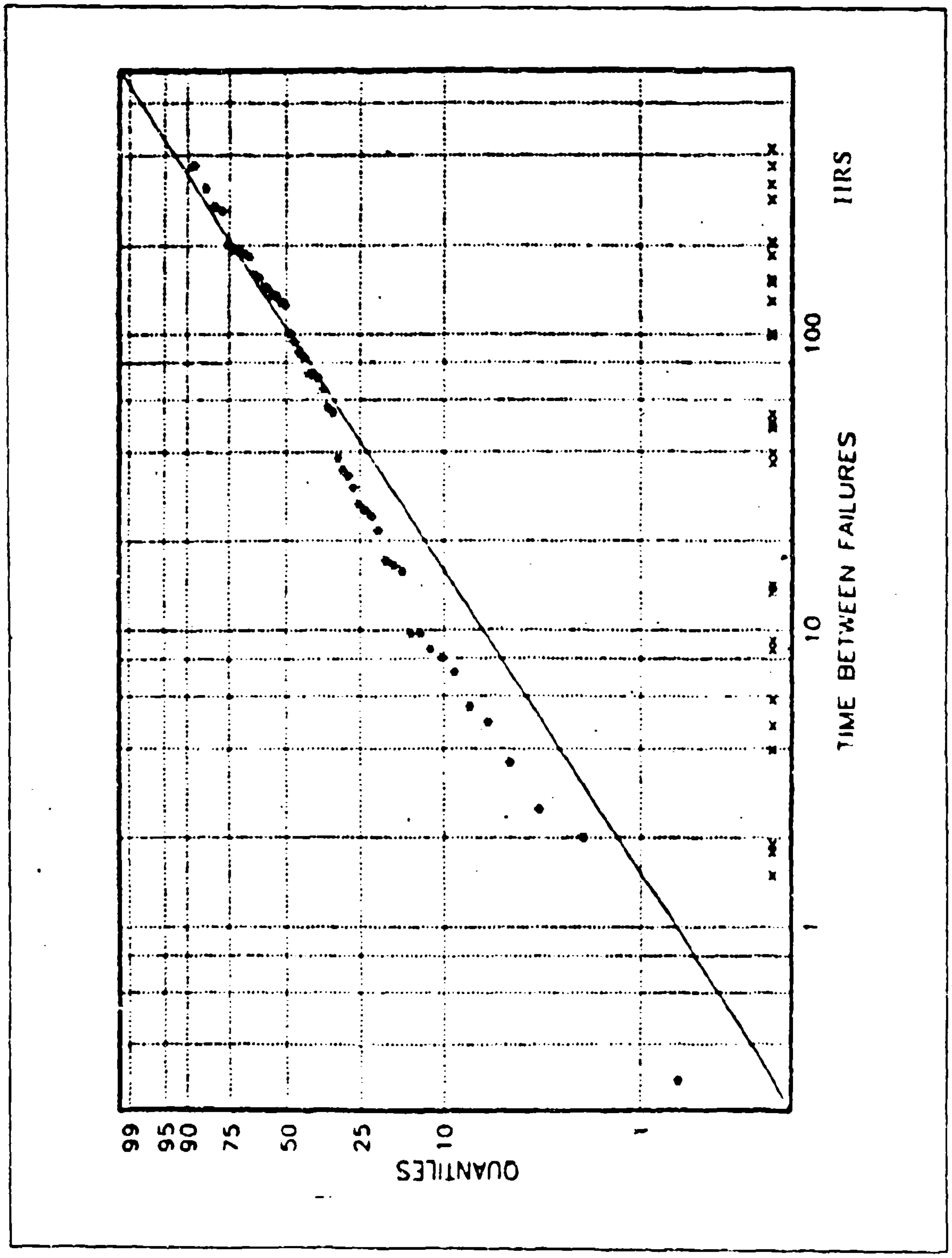

Figure F.3 Exponential Quantile-Quantile Plot: WUC 632Z100.

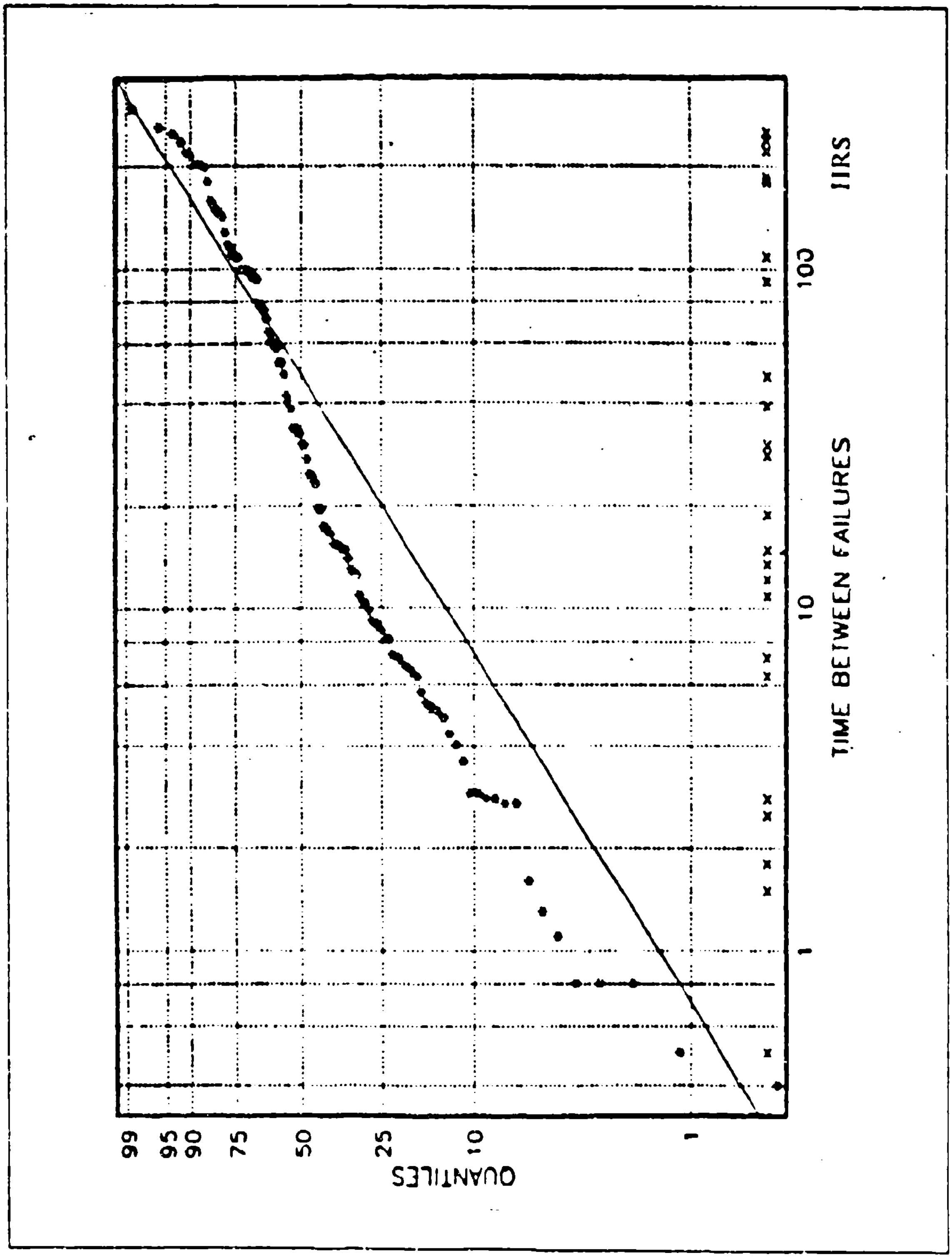

Figure F.4 Exponential Quantile-Quantile Plot: WUC 74A1500.

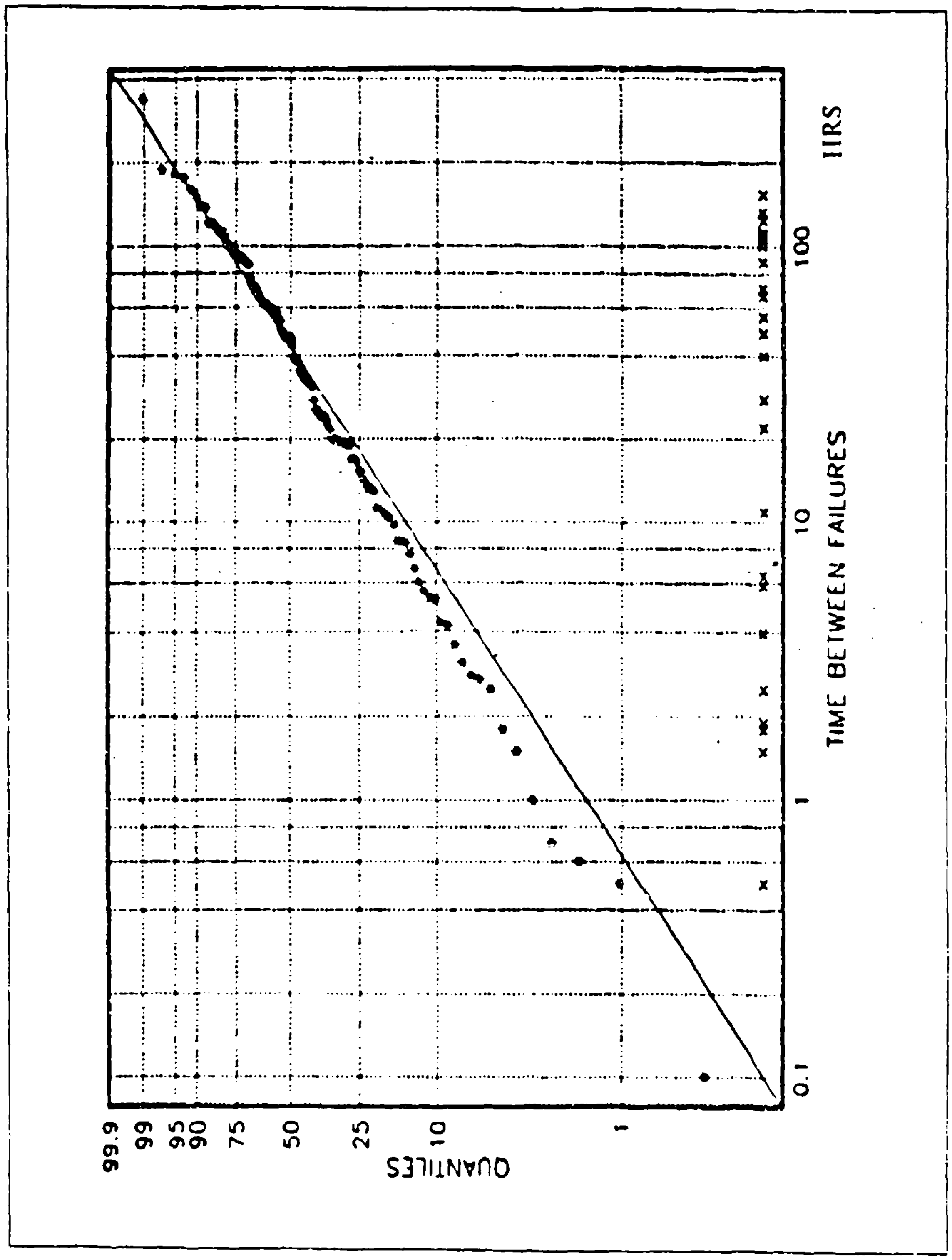

Figure F.5 Exponential Quantile-Quantile Plot: WUC 74A5M00.

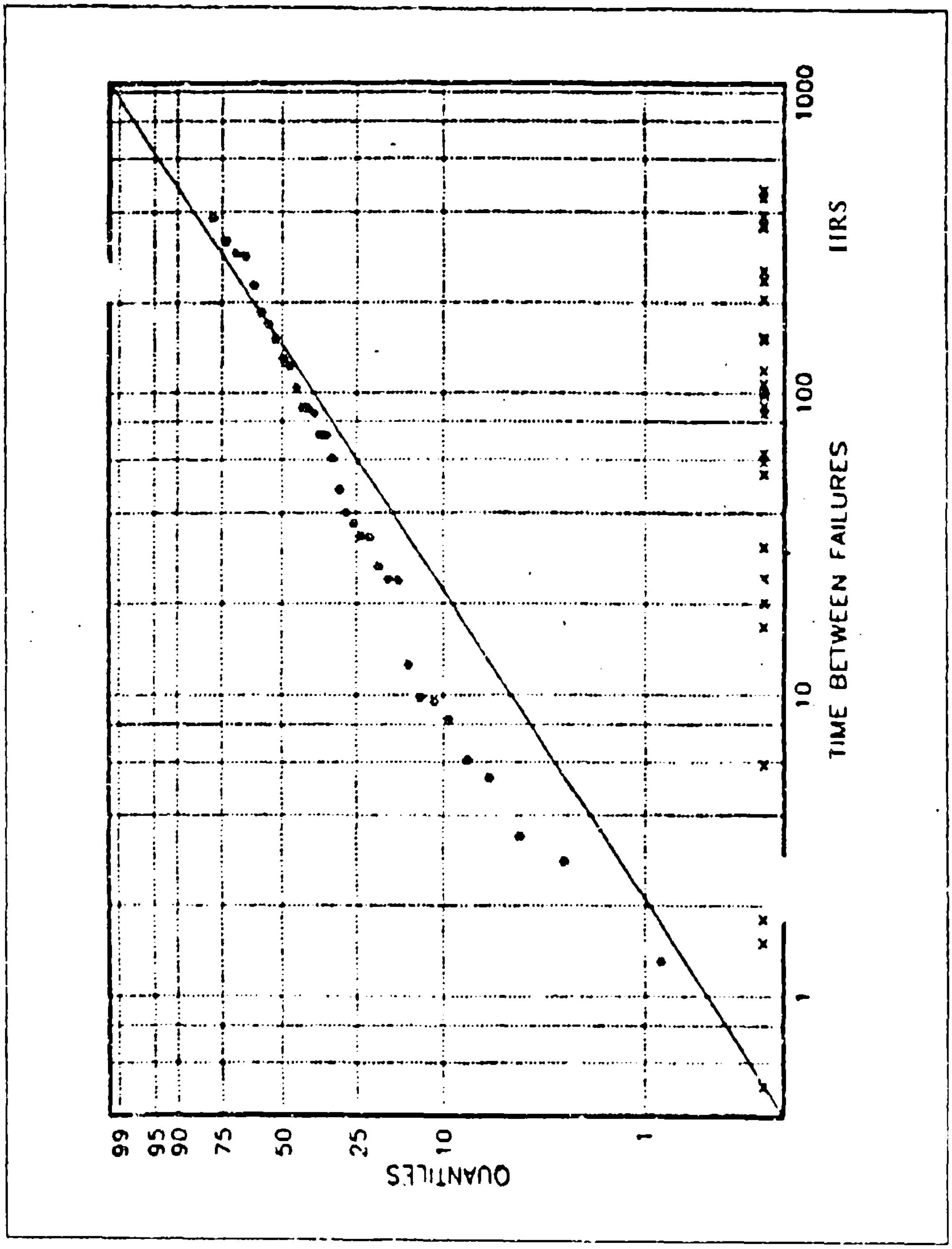

Figure F.6 Exponential Quantile-Quantile Plot: WUC 5772200.

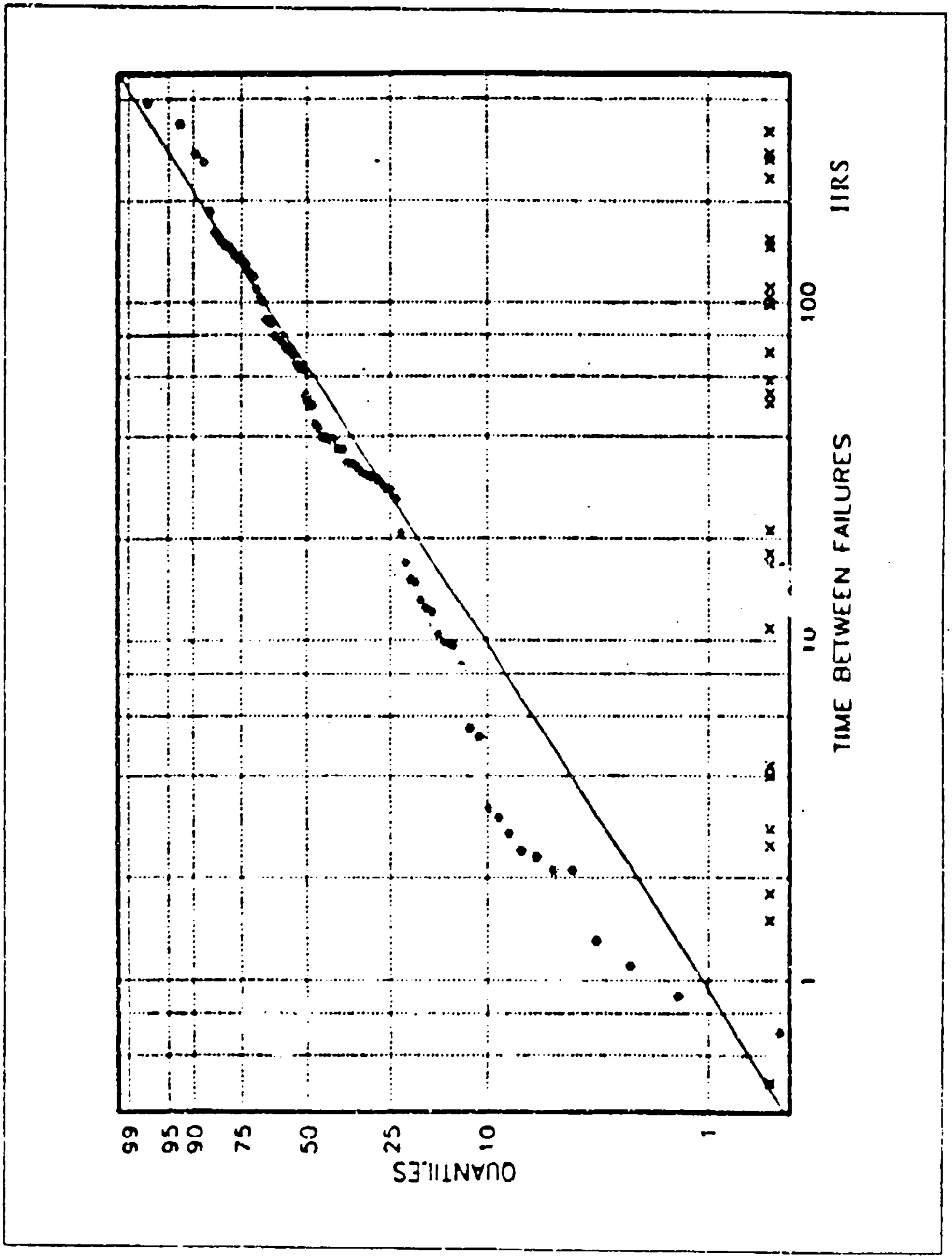

Figure F.7 Exponential Quantile-Quantile Plot: WUC 691S100.

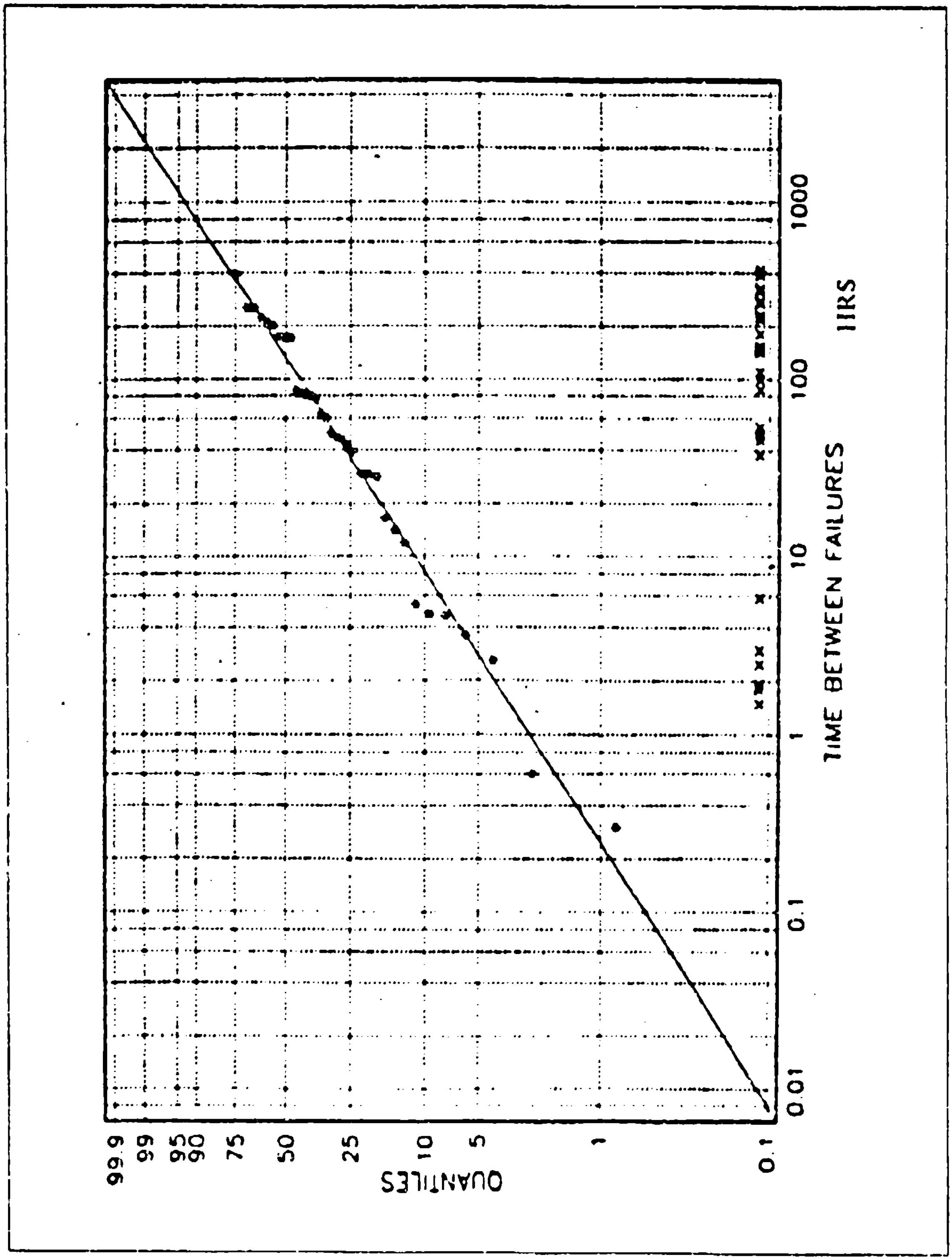

Figure F.8 Weibull Quantile-Quantile Plot: WUC 46X1600.

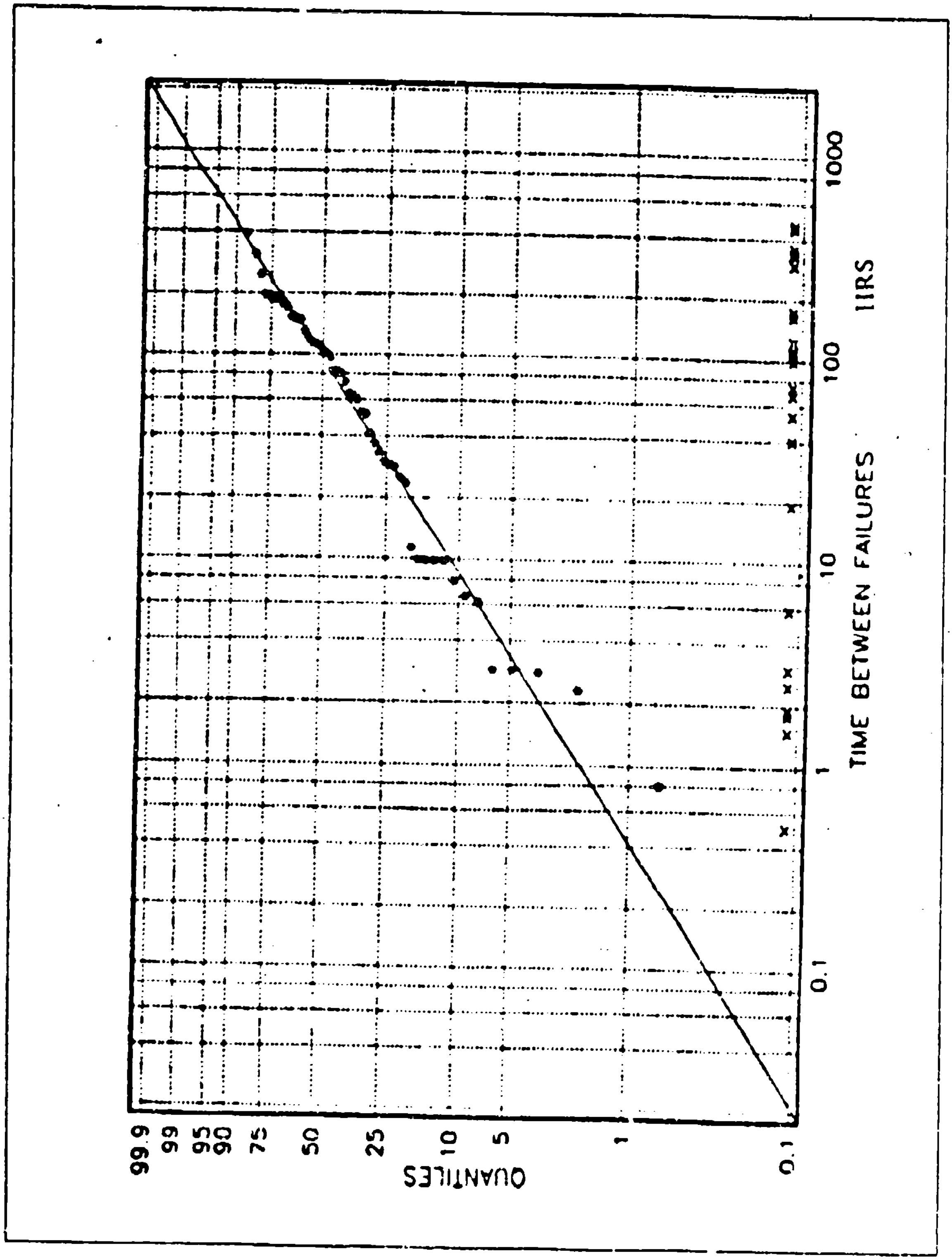

Figure F.9 Weibuli Quantile-Quantile Plot: WUC 4622100.

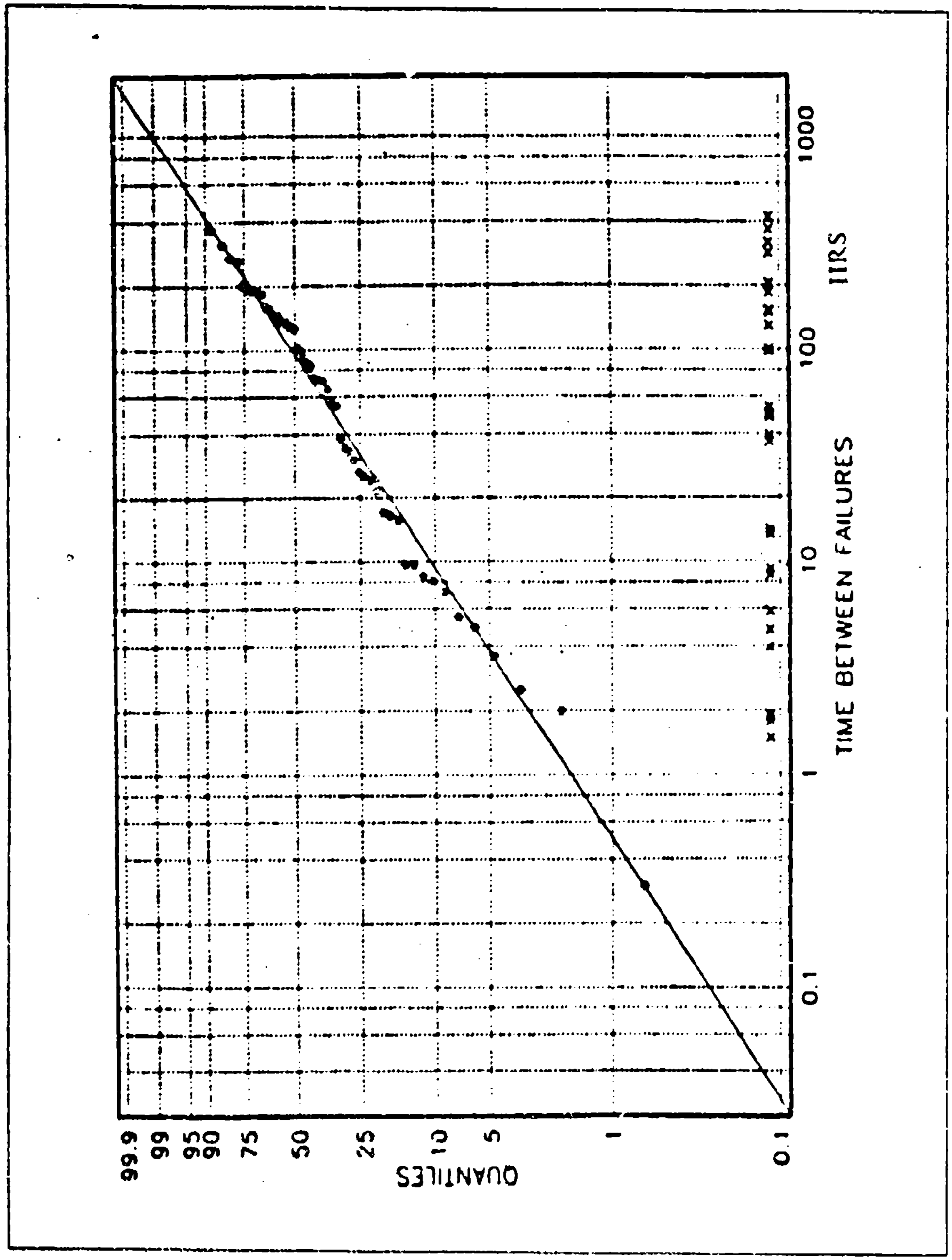

Figure F.10 Weibull Quantile-Quantile Plot: WUC 632Z100.

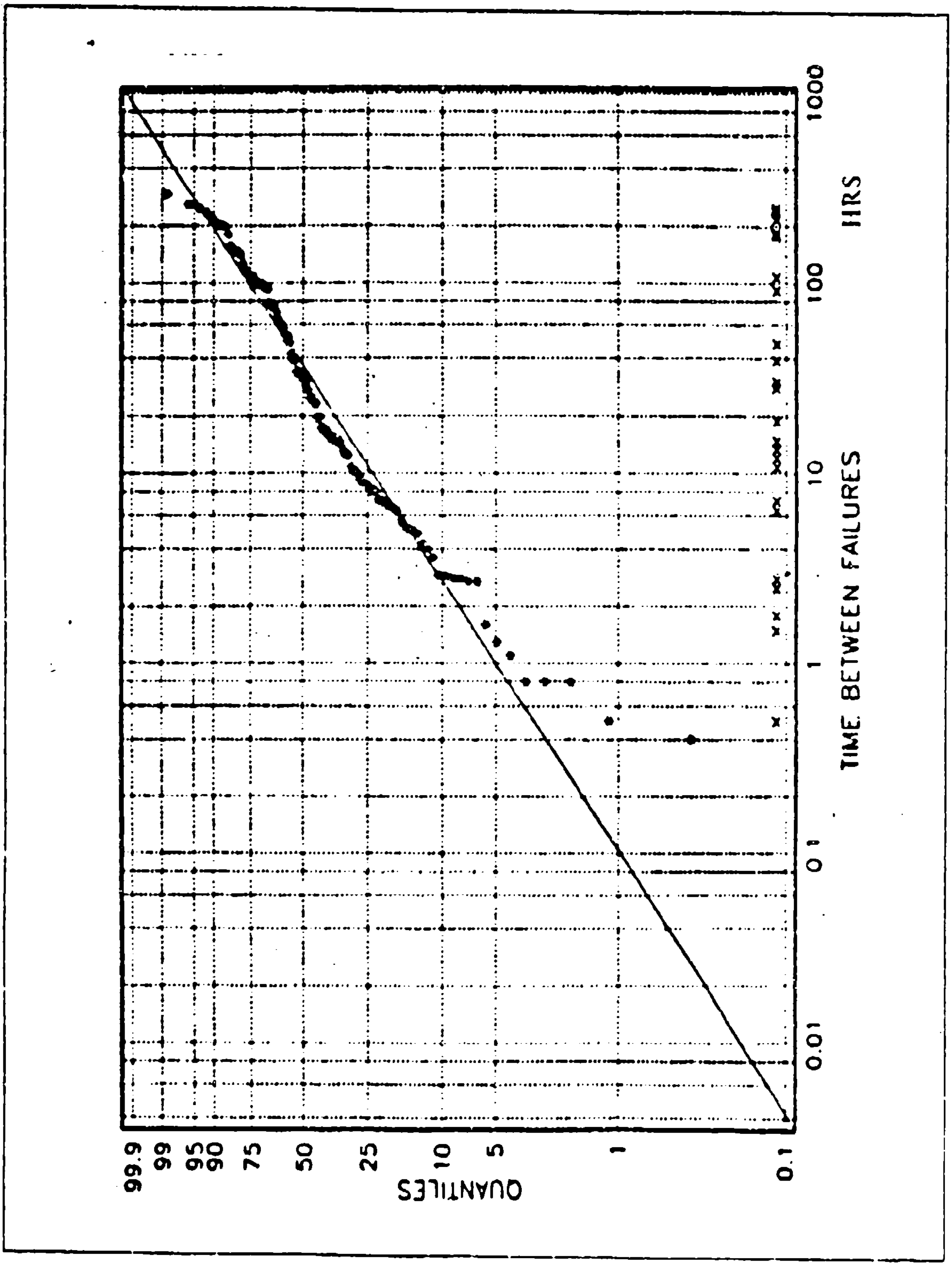

Figure F.11 Weibull Quantile-Quantile Plot: WUC 74A1500.

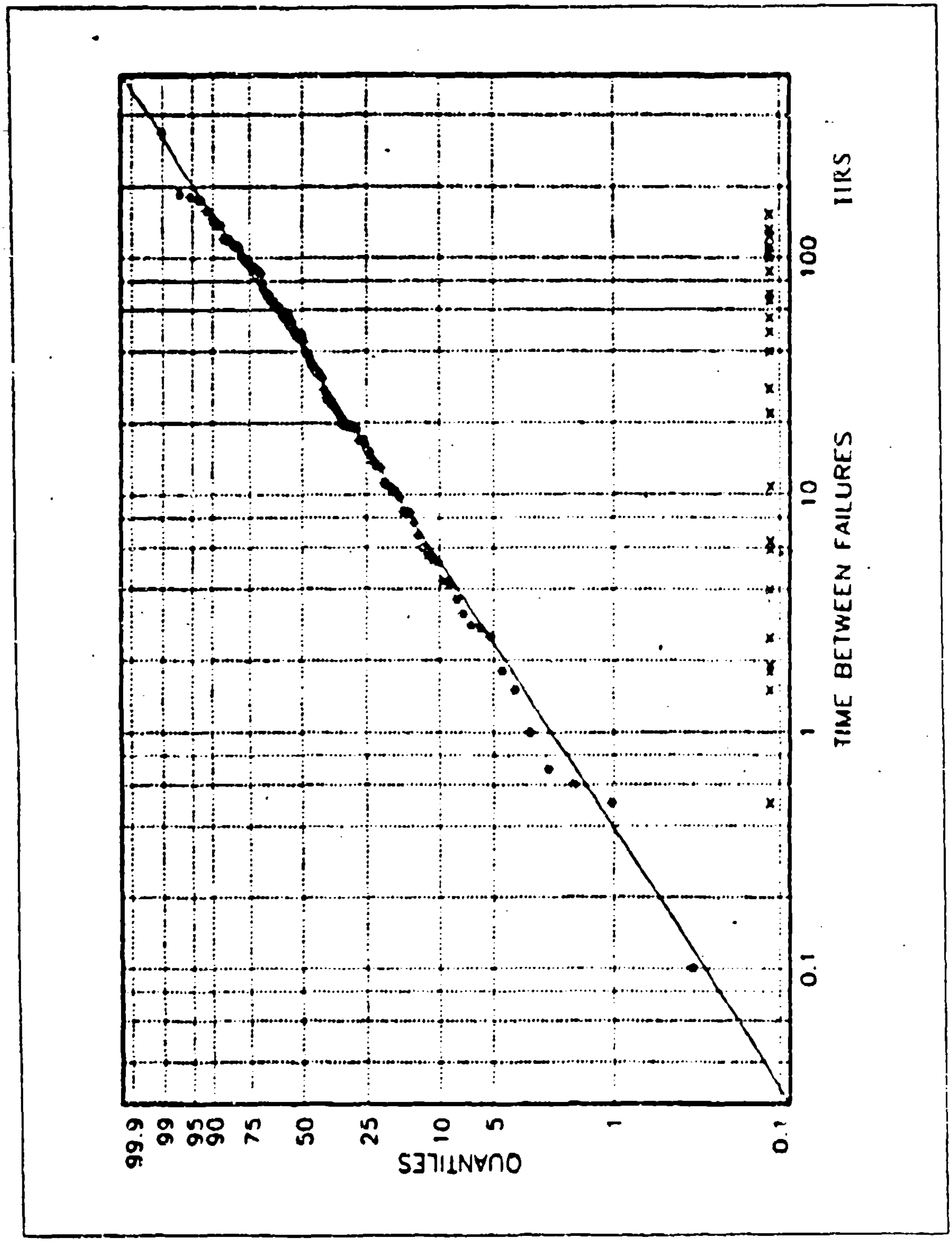

Figure F.12 Weibull Quantile-Quantile Plot: WUC 74A5M00.

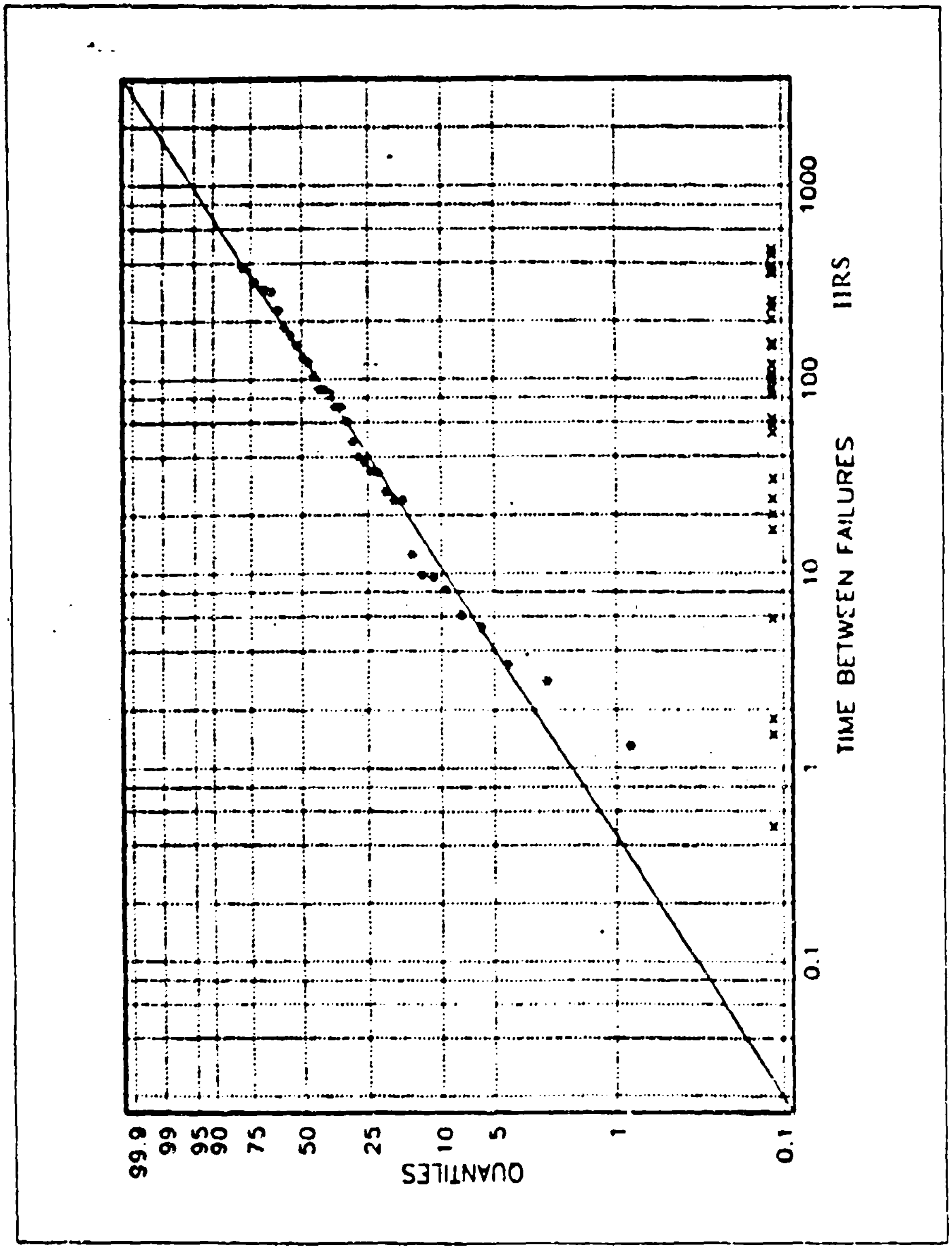

Figure F.13 Weibull Quantile-Quantile Plot: WUC 5772200.

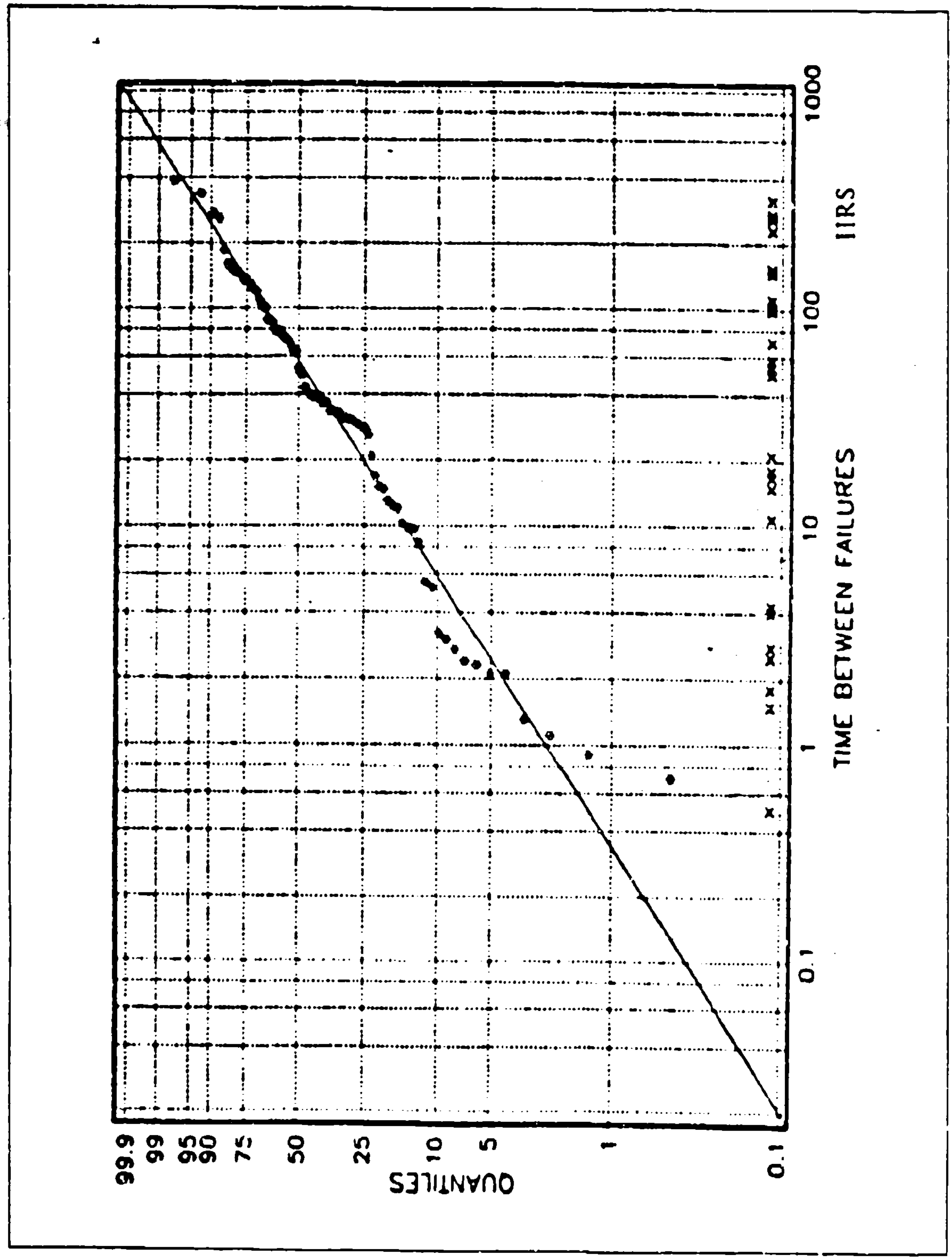

Figure F.14 Weibull Quantile-Quantile Plot: WUC 6918100.

APPENDIX G
DUAL PARAMETER OPTIMIZER

1. QUASI-NEWTON METHOD

The Quasi-Newton method was used to determine the maximum likelihood estimates for the Weibull and Mixed models. This method was derived from the Newton's method through the Taylor's expansion of the objective function, $f(x)$, where x is the $n \times 1$ column vector containing values for the n parameters being estimated. The Newton method uses the quadratic approximation to $f(x)$, equation G.1, evaluated at $x^{(k)}$ to iteratively determine an optimal solution.

$$f(x) \sim f(x^{(k)}) + \nabla f(x^{(k)})^T \Delta x + .5 \Delta x^T \nabla^2 f(x^{(k)}) \Delta x. \qquad \text{(eqn G.1)}$$

The next point in the sequence is found by forcing the gradient of the quadratic approximation (equation G.2) to zero and solving for $x^{(k+1)}$.

$$\nabla f(x) = \nabla f(x^{(k)}) + \nabla^2 f(x^{(k)}) \Delta x = 0 \qquad \text{(eqn G.2)}$$

Allowing Δx to equal $x^{(k+1)}$-$x^{(k)}$, equation G.2 can be solved for $x^{(k+1)}$.

$$x^{(k+1)} = x^{(k)} - \nabla^2 f(x^{(k)})^{-1} \nabla f(x^{(k)}) \qquad \text{(eqn G.3)}$$

The two-parameter optimizer displayed in Appendix G did not use the Newton method discussed above because the technique can be unreliable for nonquadratic functions. Instead, a more robust Quasi-Newton method was used. The major difference between the two methods is that the Quasi-Newton does not require use of the inverse Hessian matrix $\nabla^2 f(x)^{-1}$. Instead, it uses an $n \times n$ "A" matrix to estimate the inverse Hessian matrix. The $A^{(k)}$ matrix was iteratively updated using the Broyden-Fletcher-Shanno method, and should be a fair approximation to $\nabla^2 f(x^*)^{-1}$, when x^* is the optimal solution. Substituting $A^{(k)}$ for $\nabla^2 f(x^{(k)})^{-1}$ in equation G.3 results in the Quasi-Newton method for the determination of the next point in the sequence. This iterative process continues until the objective function is maximized. [Ref. 12: pp. 112-114]

2. FORTRAN PROGRAM

```
*      COMPUTES MAXIMUM LIKELIHOOD ESTIMATES FOR TWO PARAMETER
*      MODELS USING QUASI-NEWTON METHOD.

       IMPLICIT REAL*8 (A-H,O-Z)
       COMMON/RL/ALPHA,FVAL,UFVAL,DIR,FTNF,FTF,GRAD,PARAM
       COMMON/INT/CONFLG,COUNT,TFLAG,CENSOR,NF
       DIMENSION CENSOR(200),NF(200),DIR(2),FTNF(200)
       DIMENSION FTF(200),GRAD(4),PARAM(4)
       DIMENSION A(4),A1(4),A2(4),DELP(2),DELG(2)
       INTEGER CENSOR,CONFLG,COUNT,TFLAG

* INITIALIZE 'A' - APPROXIMATION FOR INVERSE HESSIAN
       DATA A(1)/1.0D0/A(2)/0.0D0/A(3)/0.0D0/A(4)/1.0D0/ITER/0/IRES/21/

       CONFLG=0
       DIR(1)=-1.0D0
       DIR(2)=-1.0D0

* INPUT DATA
       CALL INPUT
       WRITE (6,*) 'COUNT',COUNT

* OUTPUT STARTING VALUES
       WRITE (6,490)
       WRITE (6,500) (PARAM(I),I=1,2)

* COMPUTE LOGLIKELIHOOD VALUE FOR INITIAL STARTING ESTIMATES
       CALL LLIKE(1)
       FV=-1*FVAL
       WRITE(6,510) FV

* CHECK FOR CONVERGENCE
       CALL CONCHK(1)

       IF (CONFLG.EQ.1) GO TO 1000

1      ITER=ITER+1
       WRITE (6,520) ITER

* IF ITERATIONS EQUAL FIFTY, PROGRAM IS TERMINATED EARLY
       IF (ITER.GE.50) THEN
           WRITE (6,560)
           GO TO 1000
       END IF

* IF IRES=1, OBJECTIVE FUNCTION IS NO LONGER IMPROVING AND GRADIENT
* IS NOT EQUAL TO ZERO. THE "A" MATRIX IS RESET TO THE IDENTITY MATRIX.
       IF (ITER.EQ.IRES) THEN
           A(1)=1.0D0
           A(2)=0.0D0
           A(3)=0.0D0
           A(4)=1.0D0
           IRES=IRES+20
       END IF

* COMPUTE DIRECTION OF IMPROVEMENT
       DIR(1)=-1*(A(1)*GRAD(1)+A(2)*GRAD(2))
       DIR(2)=-1*(A(3)*GRAD(1)+A(4)*GRAD(2))

* COMPUTE LINE STEP SIZE (ALPHA)
       CALL LINE

* COMPUTE DIFFERENCE BETWEEN PARAM(K+1) AND PARAM(K), AND
* DIFFERENCE BETWEEN GRAD(K+1) AND GRAD(K)
       DO 10 I=1,2
           DELP(I)=PARAM(I+2)-PARAM(I)
           DELG(I)=GRAD(I+2)-GRAD(I)
```

```
10    CONTINUE

* UPDATE 'A' MATRIX
      C1=DELP(1)*DELG(1)+DELP(2)*DELG(2)

      A1(1)=DELP(1)*DELP(1)/C1
      A1(2)=DELP(1)*DELP(2)/C1
      A1(3)=DELP(1)*DELP(2)/C1
      A1(4)=DELP(2)*DELP(2)/C1

      C2=1-((DELP(1)*DELG(1))/C1)
      C3=-1*((DELP(1)*DELG(2))/C1)
      C4=-1*((DELP(2)*DELG(1))/C1)
      C5=1-((DELP(2)*DELG(2))/C1)

      A2(1)=C2*(A(1)*C2+A(2)*C3)+C3*(A(3)*C2+A(4)*C3)
      A2(2)=C2*(A(1)*C4+A(2)*C5)+C3*(A(3)*C4+A(4)*C5)
      A2(3)=C4*(A(1)*C2+A(2)*C3)+C5*(A(3)*C2+A(4)*C3)
      A2(4)=C4*(A(1)*C4+A(2)*C5)+C5*(A(3)*C4+A(4)*C5)

      DO 20 I=1,4
         A(I)=A2(I)+A1(I)
20    CONTINUE

* UPDATE OLD PARAMETER ESTIMATES AND GRADIENTS TO NEW PARAMETER
* ESTIMATES AND GRADIENTS

      DO 25 I=1,2
         PARAM(I)=PARAM(I+2)
         GRAD(I)=GRAD(I+2)
25    CONTINUE

* OUTPUT UPDATED PARAMETERS
      WRITE (6,500) (PARAM(I),I=1,2)

* OUTPUT FUNCTIONAL VALUE
      FV=-1*FVAL
      WRITE(6,510) FV

* CHECK FOR CONVERGENCE
      CALL CONCHK(1)
      IF (CONFLG.EQ.1) GO TO 1000

* START NEW ITERATION
      GO TO 1

* PROGRAM TERMINATION
1000  WRITE (6,530)
      WRITE (6,540)
      WRITE (6,550) A

* OUTPUT FORMAT
490   FORMAT ('OSTARTING VALUES:',/,' ----------------')
500   FORMAT ('OPHAT IS: ',F15.8,/,' LHAT IS: ',F15.8)
510   FORMAT ('OLOGLIKELIHOOD VALUE IS: ',F13.6)
520   FORMAT ('OITERATION: ',I3,/,' ----------')
530   FORMAT ('OPROGRAM COMPLETION')
540   FORMAT ('OFINAL "A" MATRIX IS:')
550   FORMAT ('0',2(F15.8),/,' ',2(F15.8))
560   FORMAT ('OUNSUCCESSFUL TERMINATION: ITERATIONS = 50')

      STOP
      END

      SUBROUTINE INPUT
```

```
* READS FLIGHT AND MAINTENANCE RECORDS USED IN THE CALCULATION
* OF MIXED MODEL MAXIMUM LIKELIHOOD ESTIMATES FOR PHAT AND
* LAMBDA HAT

      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/RL/ALPHA,FVAL,UFVAL,DIR,FTNF,FTF,GRAD,PARAM
      COMMON/INT/CONFLG,COUNT,TFLAG,CENSOR,NF
      DIMENSION CENSOR(200),NF(200),DIR(2),FTNF(200)
      DIMENSION FTF(200),GRAD(4),PARAM(4)
      INTEGER CENSOR,CONFLG,COUNT,TFLAG

* STARTING POINTS FOR PARAM(1),PHAT, PARAM(2),LHAT, AND RECORD COUNTER
      PARAM(1)=.67D0
      PARAM(2)=.01D0
      COUNT=1

* INPUT FLIGHT DATA
5     READ (3,500,END=1000) CENSOR(COUNT),NF(COUNT),FTNF(COUNT),FTF(COUN
     CT)

      COUNT=COUNT+1
      GO TO 5

* INPUT FORMAT
500   FORMAT (I1,1X,I4,5X,F6.1,1X,F3.1)

1000  COUNT=COUNT-1
      RETURN
      END


      SUBROUTINE CONCHK(N)

* DETERMINES IF OPTIMAL SOLUTION HAS BEEN FOUND

      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/RL/ALPHA,FVAL,UFVAL,DIR,FTNF,FTF,GRAD,PARAM
      COMMON/INT/CONFLG,COUNT,TFLAG,CENSOR,NF
      DIMENSION CENSOR(200),NF(200),DIR(2),FTNF(200)
      DIMENSION FTF(200),GRAD(4),PARAM(4)
      INTEGER CENSOR,CONFLG,COUNT,TFLAG
      REAL*8 NORMG

* TERMINATE IF THE NORM OF THE GRADIENT (NORMG) IS LESS THAN THE
* THE SPECIFIED CRITICAL VALUE (CRIT)


      NORMG=((GRAD(N)*GRAD(N))+(GRAD(N+1)*GRAD(N+1)))**.5

      IF (NORMG.LE.1.) THEN
          CONFLG=1
          WRITE (6,500)
      END IF

      FTEST=ABS(UFVAL-FVAL)

      IF (FTEST.LE..000001.AND.NORMG.GT.1.) THEN
          CONFLG=1
          WRITE (6,510)
      END IF

* OUTPUT FORMAT
500   FORMAT ('0NORMAL TERMINATION - OPTIMAL FOUND')
510   FORMAT ('0PROGRAM TERMINATION - LACK OF FUNCTIONAL IMPROVEMENT')

      RETURN
      END
```

```
      SUBROUTINE LINE

*   COMPUTES MAXIMUM STEP SIZE (ALPHA)

      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/RL/ALPHA,FVAL,UFVAL,DIR,FTNF,FTF,GRAD,PARAM
      COMMON/INT/CONFLG,COUNT,TFLAG,CENSOR,NF
      DIMENSION CENSOR(200),NF(200),DIR(2),FTNF(200)
      DIMENSION FTF(200),GRAD(4),PARAM(4)
      INTEGER CENSOR,CONFLG,COUNT,TFLAG
      REAL*8 LB

      UFVAL=FVAL
      K=1
      TFLAG=0
      ALPHA=0.0D0
      ALPHAK=.0001D0
      LB=0.0D0

*   COMPUTATION OF SEARCH INTERVAL TAU. TAU IS APPROXIMATELY
*   .618 AND TAU1 IS APPROXIMATELY .382
      TAU=(5**.5-1)/2
      TAU1=1-TAU

*   BOUNDING PHASE - SWANN'S METHOD
      STEP=.00001D0
         PARAM(3)=PARAM(1)-STEP*DIR(1)
         PARAM(4)=PARAM(2)-STEP*DIR(2)
         CALL LLIKE(3)
         F1=FVAL
         PARAM(3)=PARAM(1)+STEP*DIR(1)
         PARAM(4)=PARAM(2)+STEP*DIR(2)
         CALL LLIKE(3)
         F2=FVAL
      IF (F1.GE.F2) THEN
         F1=F2
         GO TO 5
      END IF
      IF (F2.GT.F1) STEP=-1*STEP

5     ALPHA=ALPHA+(2**K)*STEP
      PARAM(3)=PARAM(1)+ALPHA*DIR(1)
      PARAM(4)=PARAM(2)+ALPHA*DIR(2)

*   COMPUTE LOGLIKELIHOOD VALUE
      CALL LLIKE(3)
      F2=FVAL

*   IF NEW VALUE IS LESS THAN PREVIOUS VALUE TERMINATE BOUNDING

      IF (F2.GT.F1) GO TO 10
      F1=F2
      K=K+1
      GO TO 5

10    UB=ALPHA

*   CONDUCT GOLDEN SECTION SEARCH

      B1=TAU1*(UB-LB)+LB
      PARAM(3)=PARAM(1)+B1*DIR(1)
      PARAM(4)=PARAM(2)+B1*DIR(2)
      CALL LLIKE(3)
      FB1=FVAL
      B2=TAU*(UB-LB)+LB
      PARAM(3)=PARAM(1)+B2*DIR(1)
      PARAM(4)=PARAM(2)+B2*DIR(2)
      CALL LLIKE(3)
```

```
      FB2=FVAL
50    IF (FB1.LT.FB2) THEN
          ALPHA=B1
          UB=B2
          B2=B1
          FB2=FB1
          B1=TAU1*(UB-LB)+LB
          PARAM(3)=PARAM(1)+B1*DIR(1)
          PARAM(4)=PARAM(2)+B1*DIR(2)
          CALL LLIKE(3)
          FB1=FVAL
          GO TO 100
      END IF
      IF (FB2.LE.FB1) THEN
          ALPHA=B2
          LB=B1
          B1=B2
          FB1=FB2
          B2=TAU*(UB-LB)+LB
          PARAM(3)=PARAM(1)+B2*DIR(1)
          PARAM(4)=PARAM(2)+B2*DIR(2)
          CALL LLIKE(3)
          FB2=FVAL
          GO TO 100
      END IF

* CHECK FOR LINE SEARCH TERMINATION
100   CALL TERM(ALPHAK)
      ALPHAK=ALPHA
      IF (TFLAG.EQ.1) GO TO 5000
      GO TO 50

5000  PARAM(3)=PARAM(1)+ALPHA*DIR(1)
      PARAM(4)=PARAM(2)+ALPHA*DIR(2)
      CALL LLIKE(3)

      RETURN
      END

      SUBROUTINE TERM(ALPHAK)

* DETERMINES IF CONVERGENCE CRITERIA IS ESTABLISHED FOR LINE
* SEARCH TERMINATION


      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/RL/ALPHA,FVAL,UFVAL,DIR,FTNF,FTF,GRAD,PARAM
      COMMON/INT/CONFLG,COUNT,TFLAG,CENSOR,NF
      DIMENSION CENSOR(200),NF(200),DIR(2),FTNF(200)
      DIMENSION FTF(200),GRAD(4),PARAM(4)
      INTEGER CENSOR,CONFLG,COUNT,TFLAG

      TFLAG=0

      CTEST=ABS((ALPHAK-ALPHA)/ALPHA)

      IF (CTEST.LE..003) TFLAG=1

      RETURN
      END

      SUBROUTINE LLIKE(N)

* COMPUTES LOGLIKELIHOOD FUNCTIONAL VALUE AND GRADIENTS FOR
* FOR THE WEIBULL MODEL

      IMPLICIT REAL*8 (A-H,O-Z)
```

```
*    INPUT FLIGHT DATA
        DO 5 I=1,COUNT

        IF (FTF(I).EQ.0.) FTF(I)=.000001
        IF (FTNF(I).EQ.0.) FTNF(I)=.000001
        C1=(FTNF(I)+FTF(I))
        C2=PARAM(2)*C1
        C3=PARAM(2)*FTNF(I)
        C4=EXP(-1*(C2**PARAM(1)))/EXP(-1*(C3**PARAM(1)))
        SUMKI1=SUMKI1+C*((((FTNF(I)**PARAM(1)-C1**PARAM(1))*C4)*((FTNF(I)*
     C     *PARAM(1)-C1**PARAM(1))*C4))/(1-C4)+(FTNF(I)**PARAM(1))*(FTNF(I
     C     )**PARAM(1))*EXP(-1*(C3**PARAM(1))))
        SUMKI2=SUMKI2+((((C3*PARAM(1)*LOG(C3))-(C2**PARAM(1)*LOG(C2)))*C4)
     C     **2)/(1-C4)+(C3**PARAM(1)*LOG(C3))**2
5      CONTINUE

        PSD1=SQRT(1/SUMKI1)
        PSD2=SQRT(1/SUMKI2)

*    CONFIDENCE INTERVALS
        LL1=PARAM(1)-1.96*PSD1
        UL1=PARAM(1)+1.96*PSD1
        LL2=PARAM(2)-1.96*PSD1
        UL2=PARAM(2)+1.96*PSD1

*    OUTPUT
        WRITE (6,100) PSD1,LL1,UL1
        WRITE (6,110) PSD2,LL2,UL2
100    FORMAT ('0PSD1: ',F6.5,' CONFIDENCE INTERVAL (',F6.5,',',F6.5,')')
110    FORMAT ('0PSD2: ',F6.5,' CONFIDENCE INTERVAL (',F6.5,',',F6.5,')')

        RETURN
        END

        SUBROUTINE LLIKE(N)

* COMPUTES LOGLIKELIHOOD FUNCTIONAL VALUE AND GRADIENTS FOR
* THE MIXED MODEL.

        IMPLICIT REAL*8 (A-H,O-Z)
        COMMON/RL/ALPHA,FVAL,UFVAL,DIR,FTNF,FTF,GRAD,PARAM
        COMMON/INT/CONFLG,COUNT,TFLAG,CENSOR,NF
        DIMENSION CENSOR(200),NF(200),DIR(2),FTNF(200)
        DIMENSION FTF(200),GRAD(4),PARAM(4)
        INTEGER CENSOR,CONFLG,COUNT,TFLAG

* ESTABLISH BOUNDARY CONDITIONS
        IF (PARAM(N).LT..0000000001) PARAM(N)=.0001
        IF (PARAM(N).GE.1.) PARAM(N)=.9999
        IF (PARAM(N+1).LT..0000000001) PARAM(N+1)=.0001

        FVAL=0.
        GRAD(N)=0.
        GRAD(N+1)=0.
        C1=1-PARAM(N)
        C2=LOG(C1)

* INPUT FLIGHT DATA
        DO 5 I=1,COUNT

* UNCENSORED RECORDS
         IF (CENSOR(I).EQ.0) THEN
            C3=EXP(-PARAM(N+1)*FTF(I))
*       FUNCTIONAL VALUE CALCULATIONS
            FVAL=FVAL-LOG(1-C3*C1)-(NF(I)-1)*C2+PARAM(N+1)*FTNF(I)
*       GRADIENT CALCULATIONS
            GRAD(N)=GRAD(N)-(C3/(1-C1*C3))+(NF(I)-1)/C1
            GRAD(N+1)=GRAD(N+1)-(FTF(I)*C1*C3)/(1-C1*C3)+FTNF(I)
```

```
        GO TO 5
      END IF

* CENSORED RECORDS
*     FUNCTIONAL VALUE CALCULATIONS
        FVAL=FVAL-NF(I)*C2+PARAM(N+1)*FTNF(I)
*     GRADIENT CALCULATIONS
        GRAD(N)=GRAD(N)+NF(I)/C1
        GRAD(N+1)=GRAD(N+1)+FTNF(I)
5     CONTINUE

      RETURN
      END
```

APPENDIX H
WEIBULL SIMULATION OF COMPONENT FAILURES

```
*     VARIABLE DEFINITIONS

*     ALPHA:WEIBULL SHAPE PARAMETER
*     AC:MATRIX USED TO STORE EACH AIRCRAFTS PREVIOUS FLIGHT HOURS
*        WITHOUT FAILURE AND UNCONDITIONAL SURVIVAL FUNCTION
*     AS:AIRCRAFT SELECTED FOR FLIGHT EVOLUTION
*     CPF:CONDITIONAL PROBABILITY OF FAILURE
*     DTIME:DEPLOYMENT TIME
*     FAIL:MATRIX USED TO STORE SIMULATED FAILURES AT SPECIFIED TIME
*     INTERVALS FOR EACH DEPLOYMENT SIMULATION
*     FT:FLIGHT TIME
*     GFC:GENERATED FAILURE CRITERIA
*     HM:HISTORICAL MEAN
*     HSD:HISTORICAL STANDARD DEVIATION
*     LHAT:MAXIMUM LIKELIHOOD ESTIMATE FOR WEIBULL SCALE PARAMETER
*     NAC:NUMBER OF AIRCRAFT USED IN SIMULATION
*     NI:NUMBER OF TIME INTERVALS TO BE USED IN SIMULATION (500 HR INT.)
*     NS:NUMBER OF SIMULATIONS TO BE RUN
*     NSEED1:SEED USED TO DETERMINE AIRCRAFT CHOSEN FOR FLIGHT
*     NSEED2:SEED USED TO DETERMINE FLIGHT DURATION
*     NSEED3:SEED USED TO HDETERMINE IF A COMPONENT FAILURE WAS OBSERVED
*     TGATE:TIME GATE
*     TIME:ACCUMULATED FLIGHT TIME
*     UCPS:UNCONDITIONAL PROBAILITY OF FAILURE

         DIMENSION AC(70,2),FAIL(20,100),EST(20,4)

         INTEGER AS
         REAL * 4 LHAT

*     INITIALIZE
         DATA LHAT/.0149/ALPHA/.8079/
         DATA NI/16/NS/100/NAC/48/HM/2.18/HSD/.9897/
         DATA NSEED1/749684/NSEED2/4683957/NSEED3/5872654/

*     COMMENCE SIMULATION(I)
         DO 1 I=1,NS
            NF=0
            TIME=0.
            TGATE=500.
            DO 5 K=1,NAC
               AC(K,1)=0.
               AC(K,2)=1.
5        CONTINUE

*     SIMULATE AND RECORD COMPONENT FAILURES USING 500 HR INTERVALS

            DO 10 J=1,NI

*     LRND AND LNORM ARE SUBROUTINES IN A RANDOM NUMBER GENERATOR PACKAGE
*     CALLED LLRANDOMII: A NON-IMSL PROGRAM AVAILABLE AT THE NAVAL
*     POSTGRADUATE SCHOOL.

*     GENERATE UNIFORM (0,1) RANDOM VARIABLE TO DETERMINE WHICH AIRCRAFT
*     WILL BE FLOWN

11             CALL LRND(NSEED1,U,1,2,0)

                 AS=(U*(NAC-1))+1.5

*     GENERATE NORMAL (0,1) RANDOM VARIABLE TO COMPUTE FLIGHT DURATION
```

```
               CALL LNORM(NSEED2,RN,1,2,0)

               FT=(RN*HSD)+HM
               IF (FT.LT..5) FT=.5
               IF (FT.GT.6.) FT=6.

*    COMPUTE THE UNCONDITIONAL PROBABILITY OF SURVIVAL FOR S+FT HRS,
*    WHERE S EQUALS THE TOTAL FLIGHT HOURS FLOWN SINCE THE LAST
*    COMPONENT FAILURE, AND FT EQUALS THE FLIGHT TIME OF THE SORTIE
*    BEING FLOWN.

               UCPS=EXP(-1*((LHAT*(AC(AS,1)+FT))**ALPHA))

               CPF=1-UCPS/AC(AS,2)

*    GENERATE UNIFORM (0,1) RANDOM VARIABLE TO DETERMINE IF COMPONENT
*    FAILURE OCCURS

               CALL LRND(NSEED3,GFC,1,2,0)

*    COMPONENT FAILURE

               IF (GFC.LT.CPF) THEN
                   NF=NF+1
                   AC(AS,1)=0.
                   AC(AS,2)=1.
                   GO TO 15
               END IF

*    COMPONENT SURVIVES

               AC(AS,1)=AC(AS,1)+FT
               AC(AS,2)=UCPS

*    UPDATE TIME

15          TIME=TIME+FT

               IF (TIME.GE.TGATE) THEN
                   FAIL(J,I)=NF
                   TGATE=TGATE+500.
                   GO TO 10
               END IF

*    NEXT AIRCRAFT SORTIE TO BE SIMULATED

               GO TO 11

*    NEXT TIME INTERVAL

10       CONTINUE

*    NEXT DEPLOYMENT

1      CONTINUE

*    CALCULATE MEAN AND STANDARD DEVIATION FOR THE NUMBER OF FAILURES
*    OBSERVED AT EACH TIME INTERVAL (MEAN=EST(J,1);STANDARD
*    DEVIATION=EST(J,2))

       DO 20 J=1,NI
           DO 25 I=1,NS
               EST(J,1)=EST(J,1)+FAIL(J,I)
               EST(J,2)=EST(J,2)+FAIL(J,I)*FAIL(J,I)
25         CONTINUE
20     CONTINUE

       DO 30 J=1,NI
```

```
      EST(J,1)=EST(J,1)/NS
      EST(J,2)=((EST(J,2)-(NS*EST(J,1)*EST(J,1)))/(NS-1))**.5
30    CONTINUE

*     95 PERCENT CONFIDENCE INTERVAL (LOWER LIMIT=EST(J,3);
*     UPPER LIMIT=EST(J,4)

      DO 35 J=1,NI
         HINT=1.96*EST(J,2)
         EST(J,3)=EST(J,1)-HINT
         EST(J,4)=EST(J,1)+HINT
35    CONTINUE

*     OUTPUT

      TIME=500.
*     WRITE (6,1000)
      DO 40 J=1,NI
         WRITE  (6,1100) TIME,EST(J,1),EST(J,2),EST(J,3),EST(J,4)
         TIME=TIME+500.
40    CONTINUE

* OUTPUT FORMAT

*1000  FORMAT ('1 TIME   MEAN     STD DEV   LOWER     UPPER')
1100  FORMAT (F5.0,4(2X,F7.3))

      STOP
      END
```

APPENDIX I
GRAPHICAL COMPARISONS

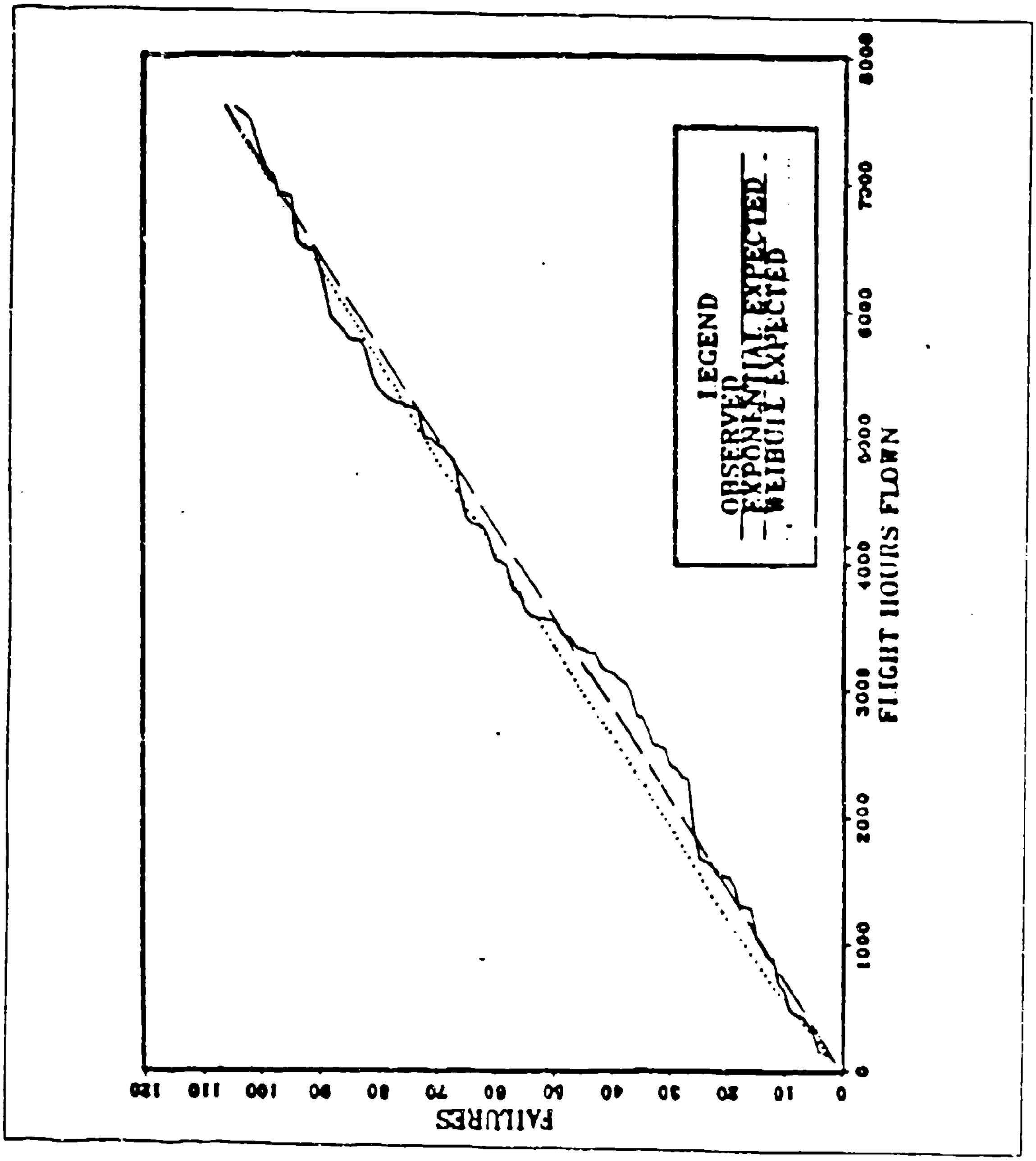

Figure I.1 Flight Hour Models: Model Set WUC:73411100.

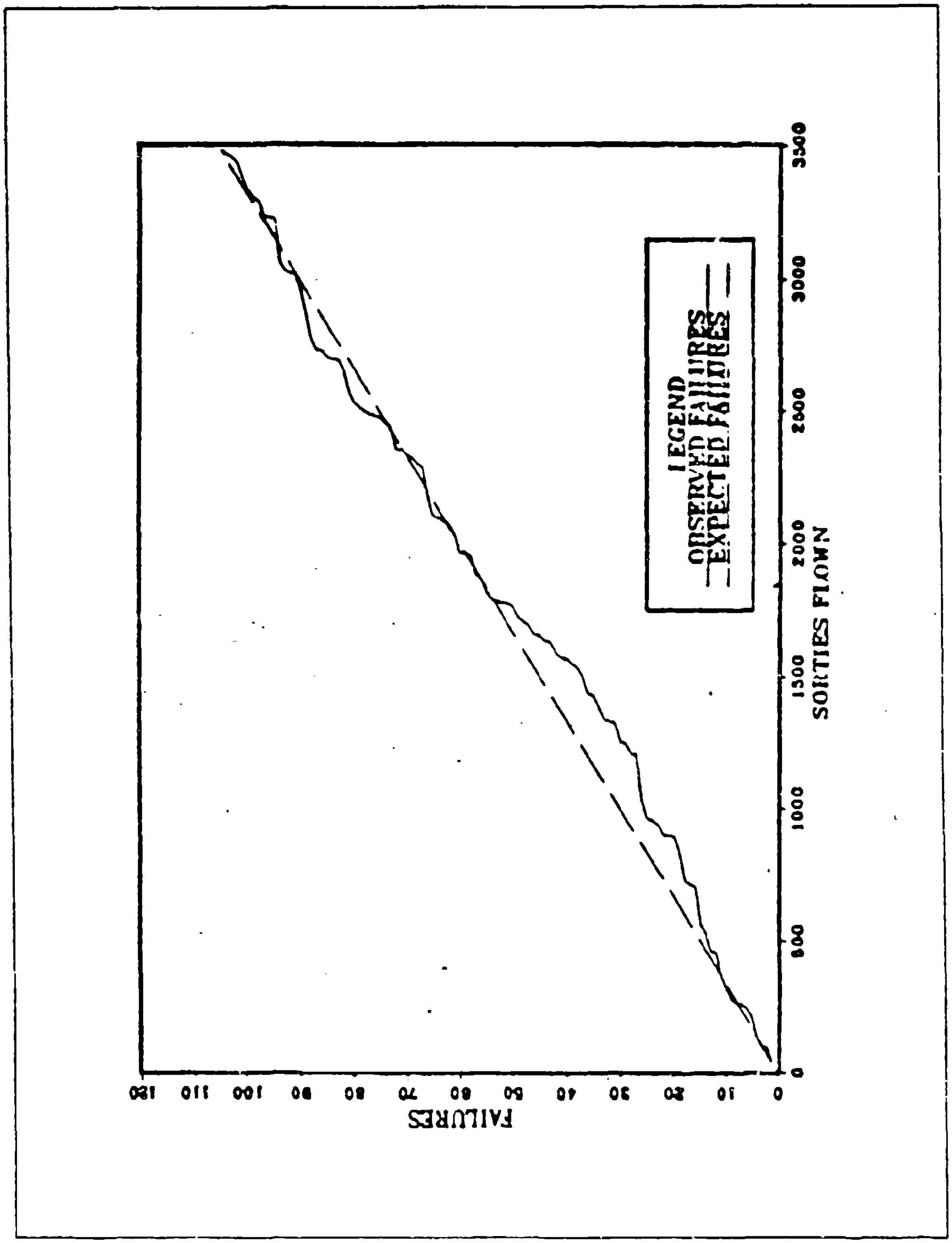

Figure 1.2   Geometric Model: Model Set WUC:734l1100.

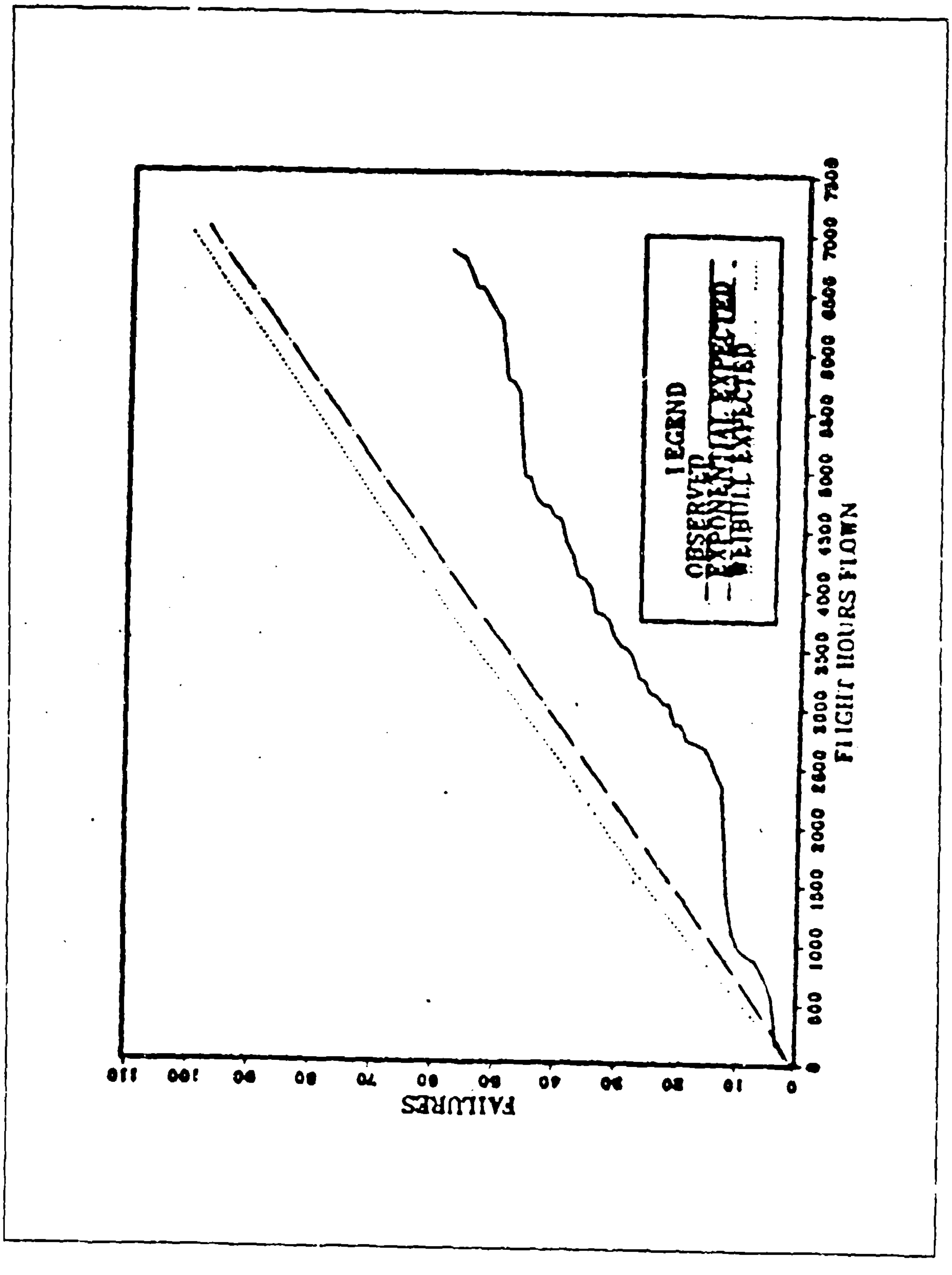

Figure I.3 Flight Hour Models: Validation Set WUC:73411100.

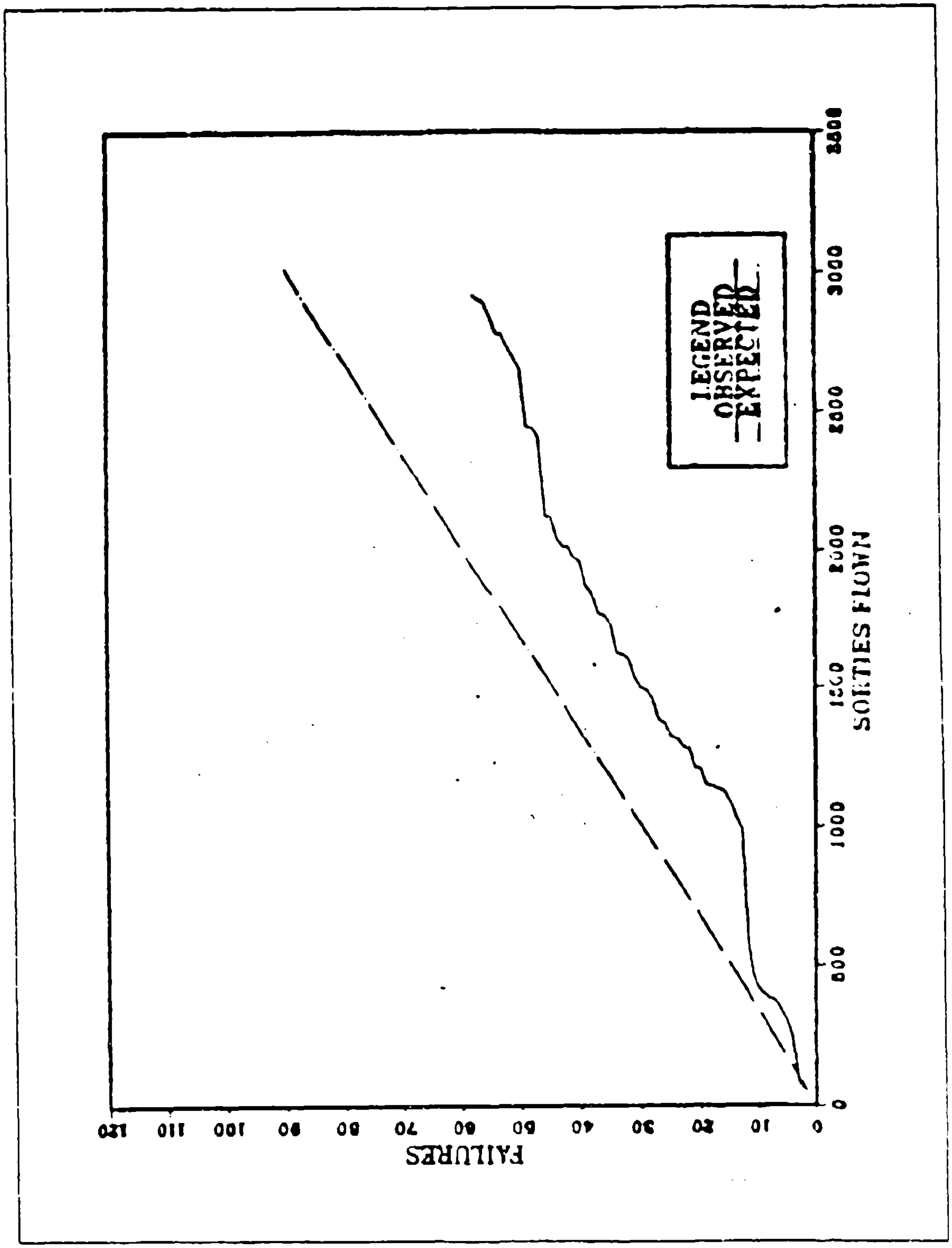

Figure I.4 Geometric Model: Validation Set WUC:734H100.

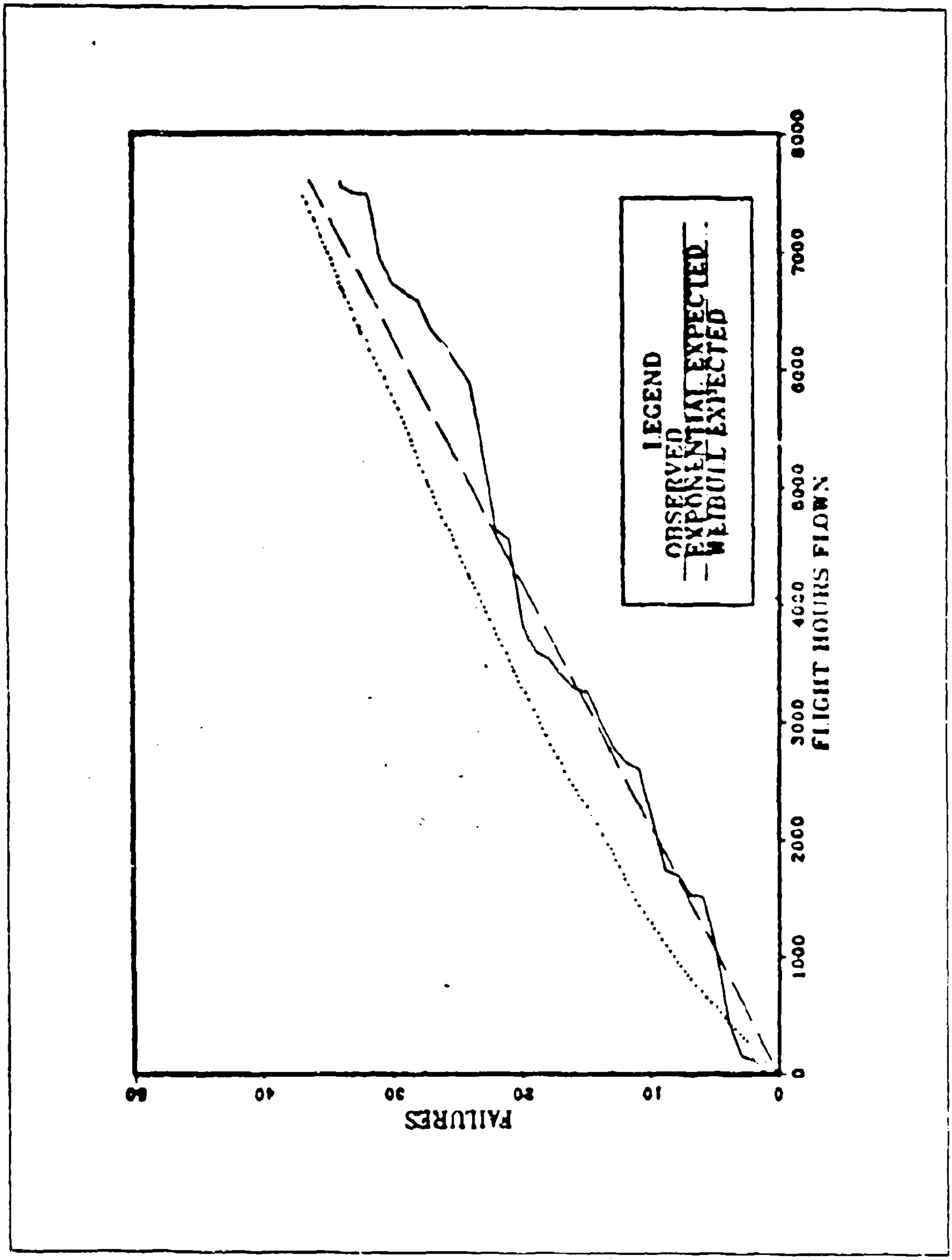

Figure I.5 Flight Hour Models: Model Set WUC:46X1600.

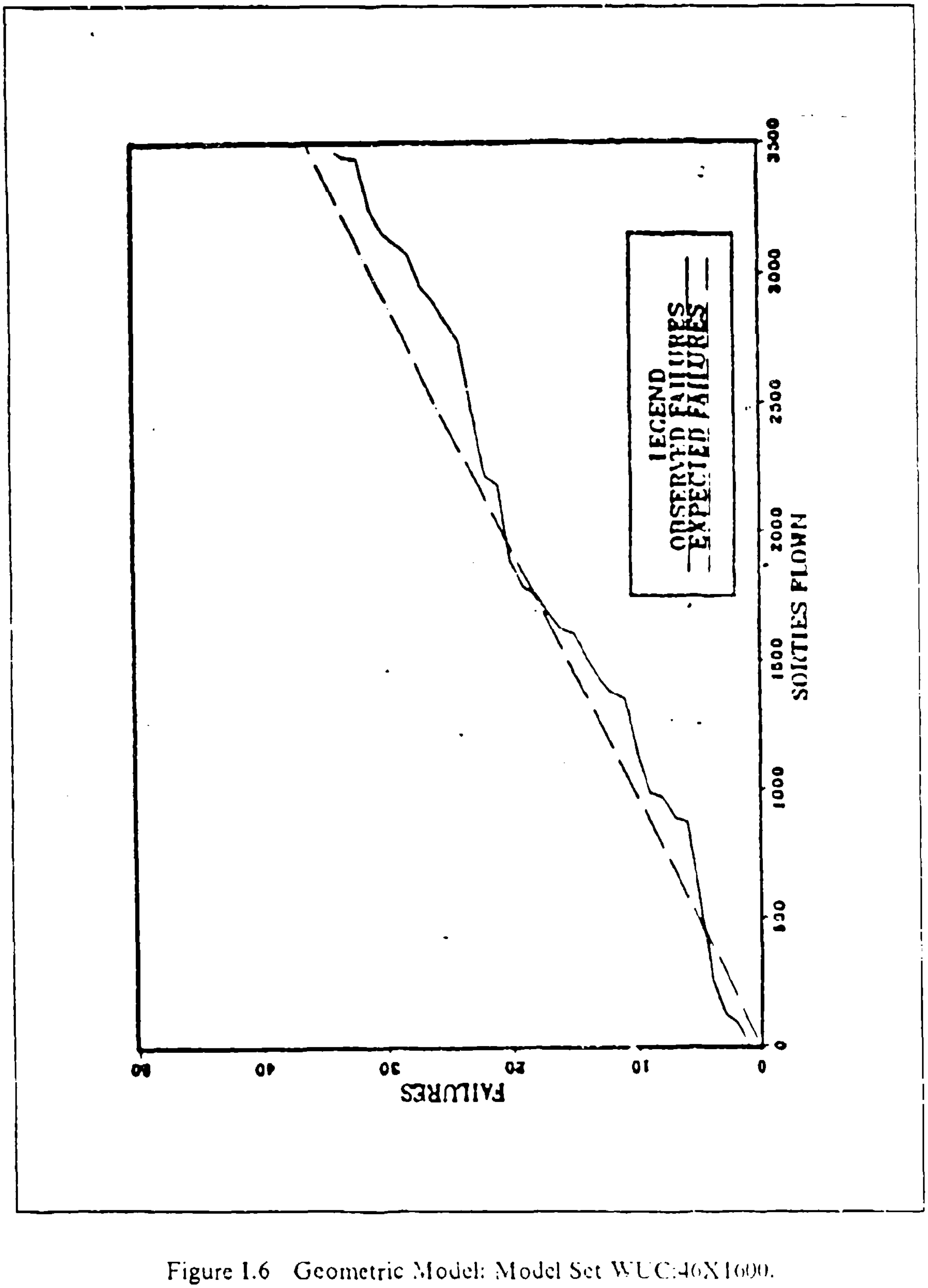

Figure I.6 Geometric Model: Model Set WUC:46X1600.

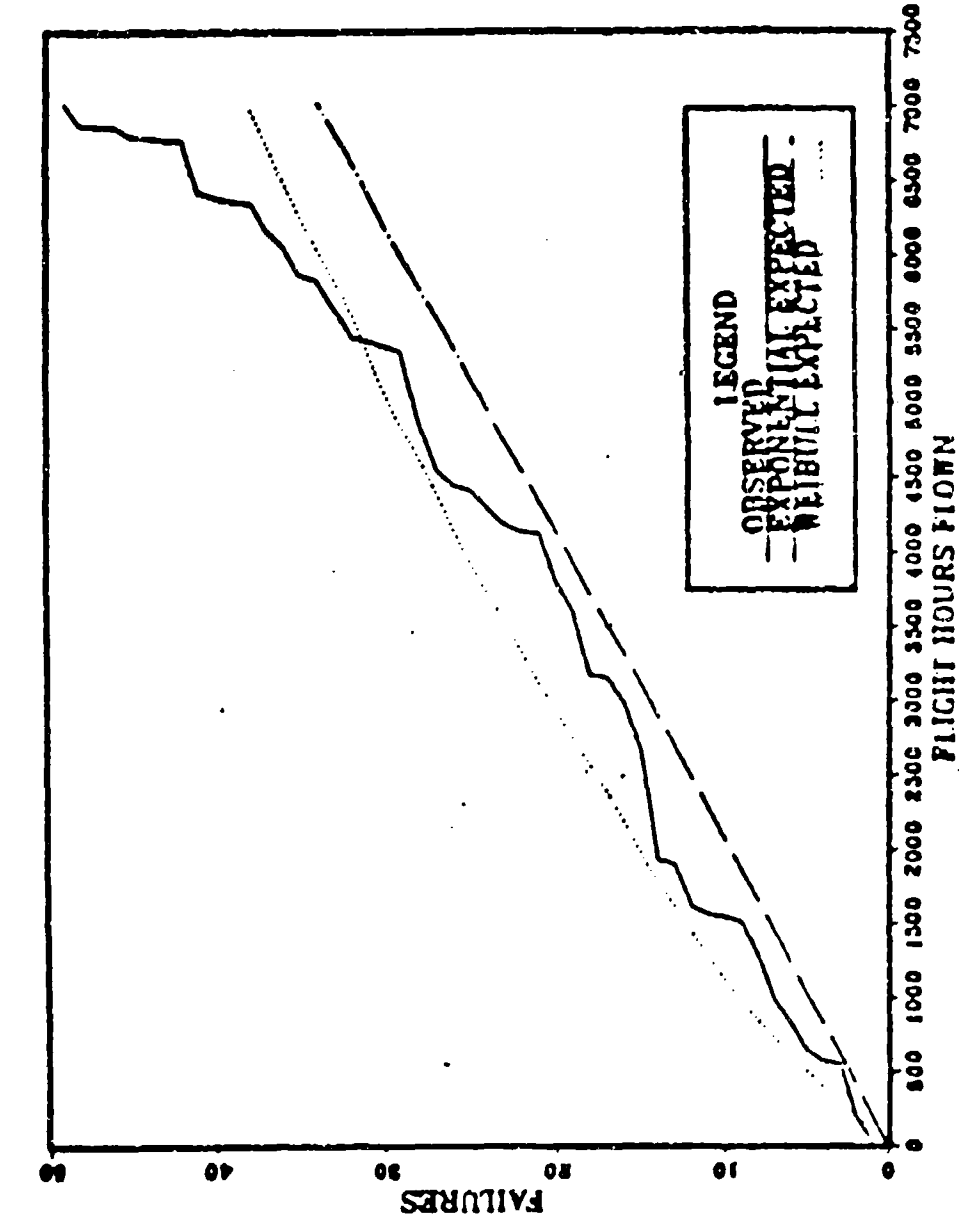

Figure I.7 Flight Hour Models: Validation Set WUC:46X1600.

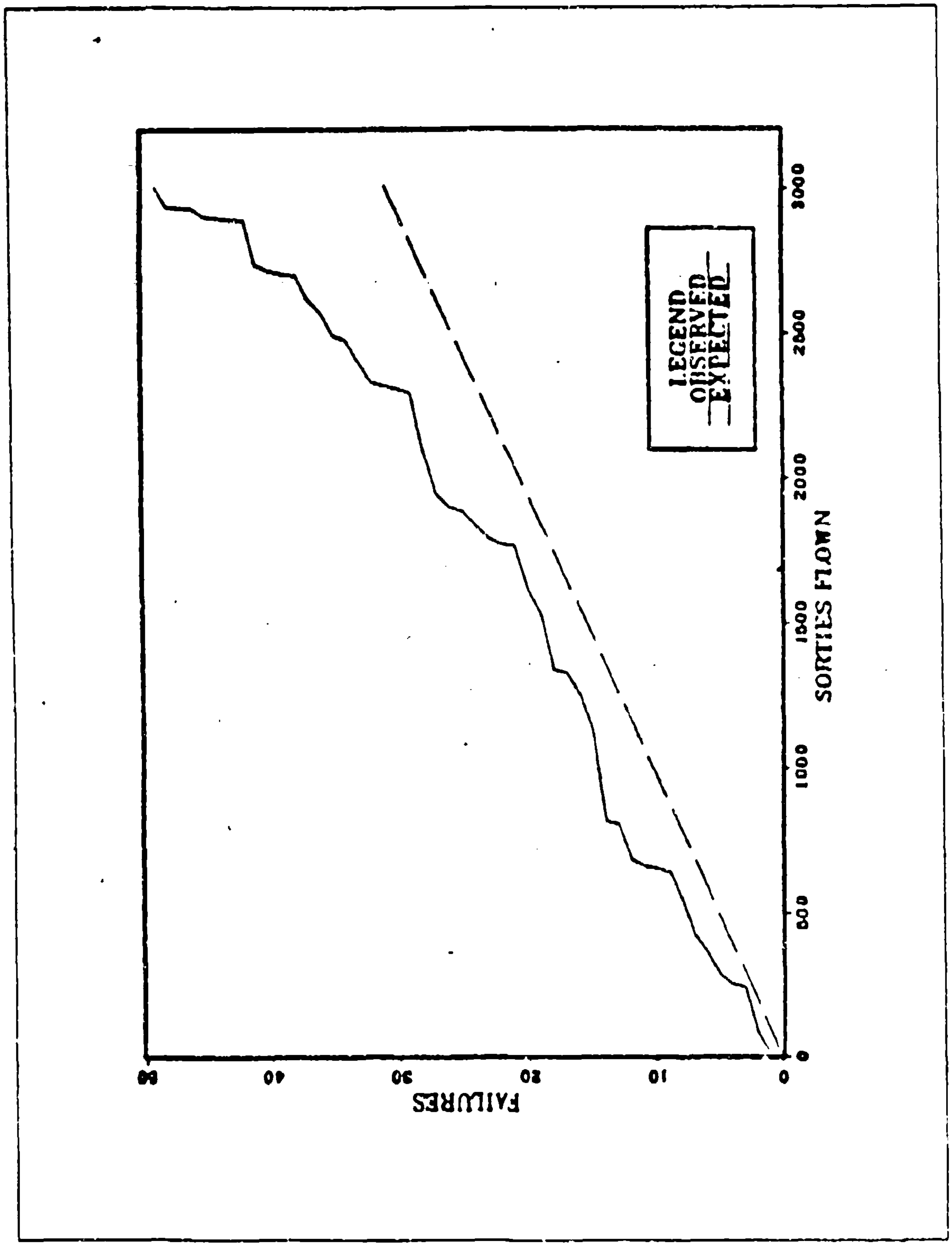

Figure I.8 Geometric Model: Validation Set WUC:46x1600.

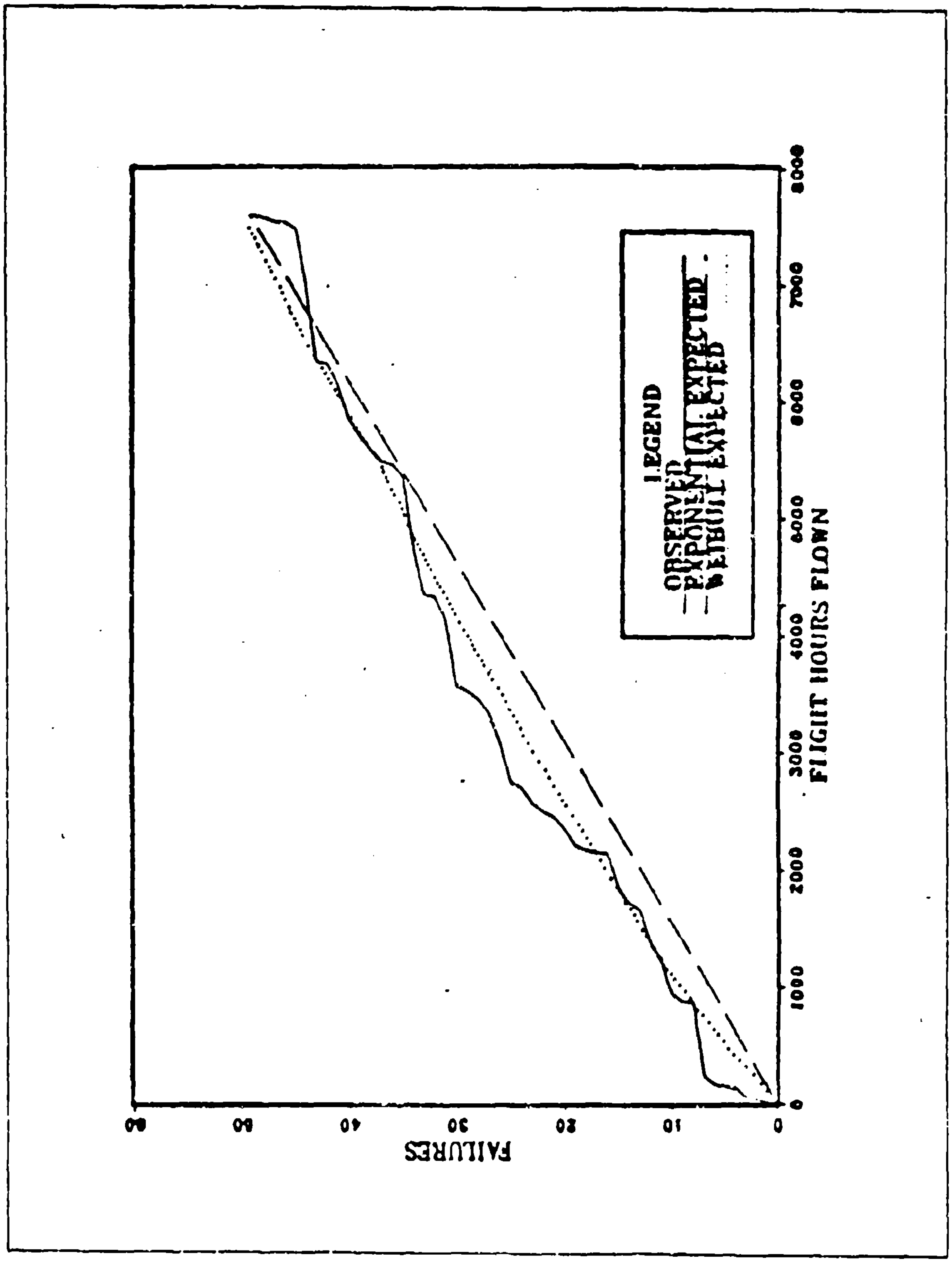

Figure I.9 Flight Hour Models: Model Set WUC:4622100.

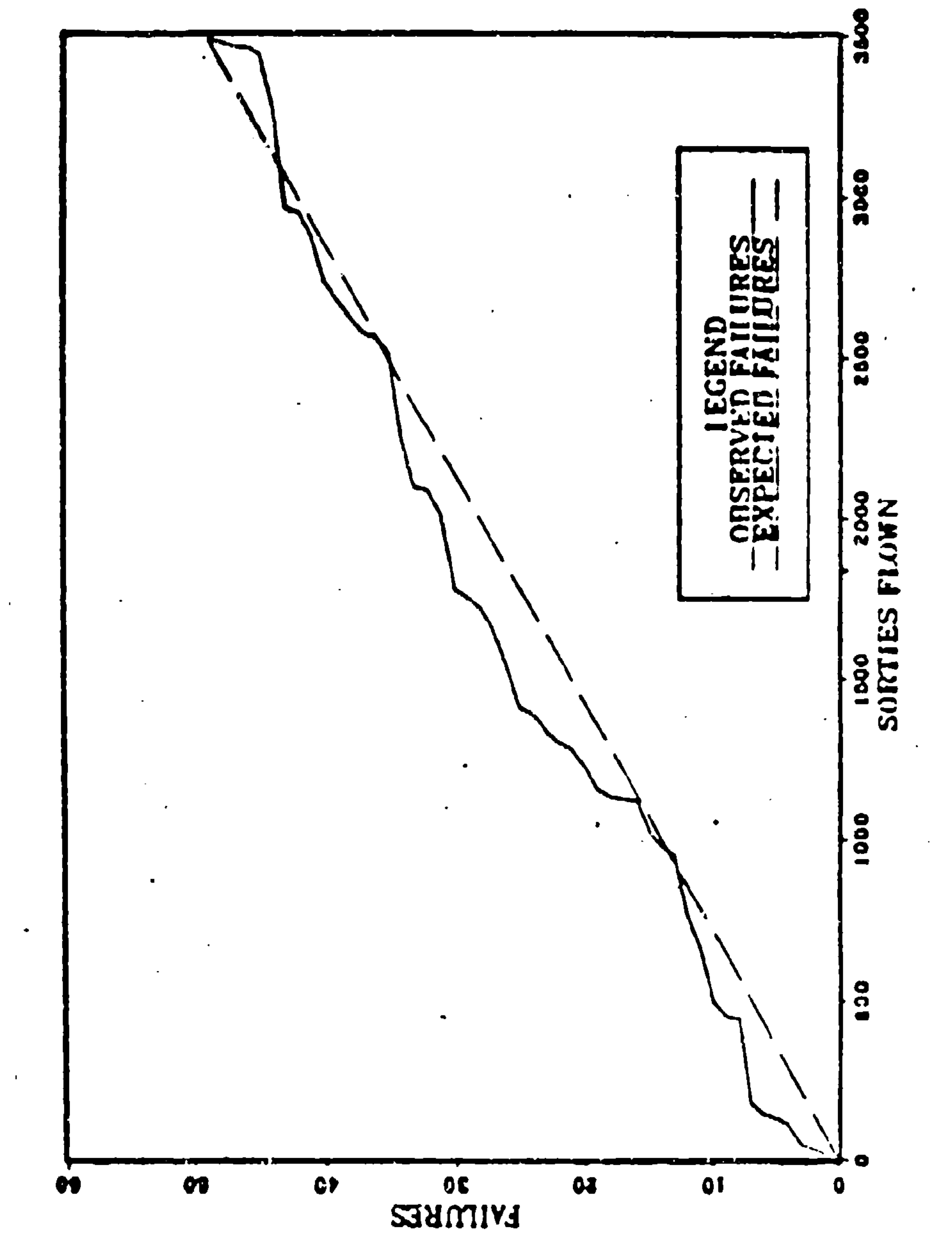

Figure I.10 Geometric Model: Model Set WUC:4622100.

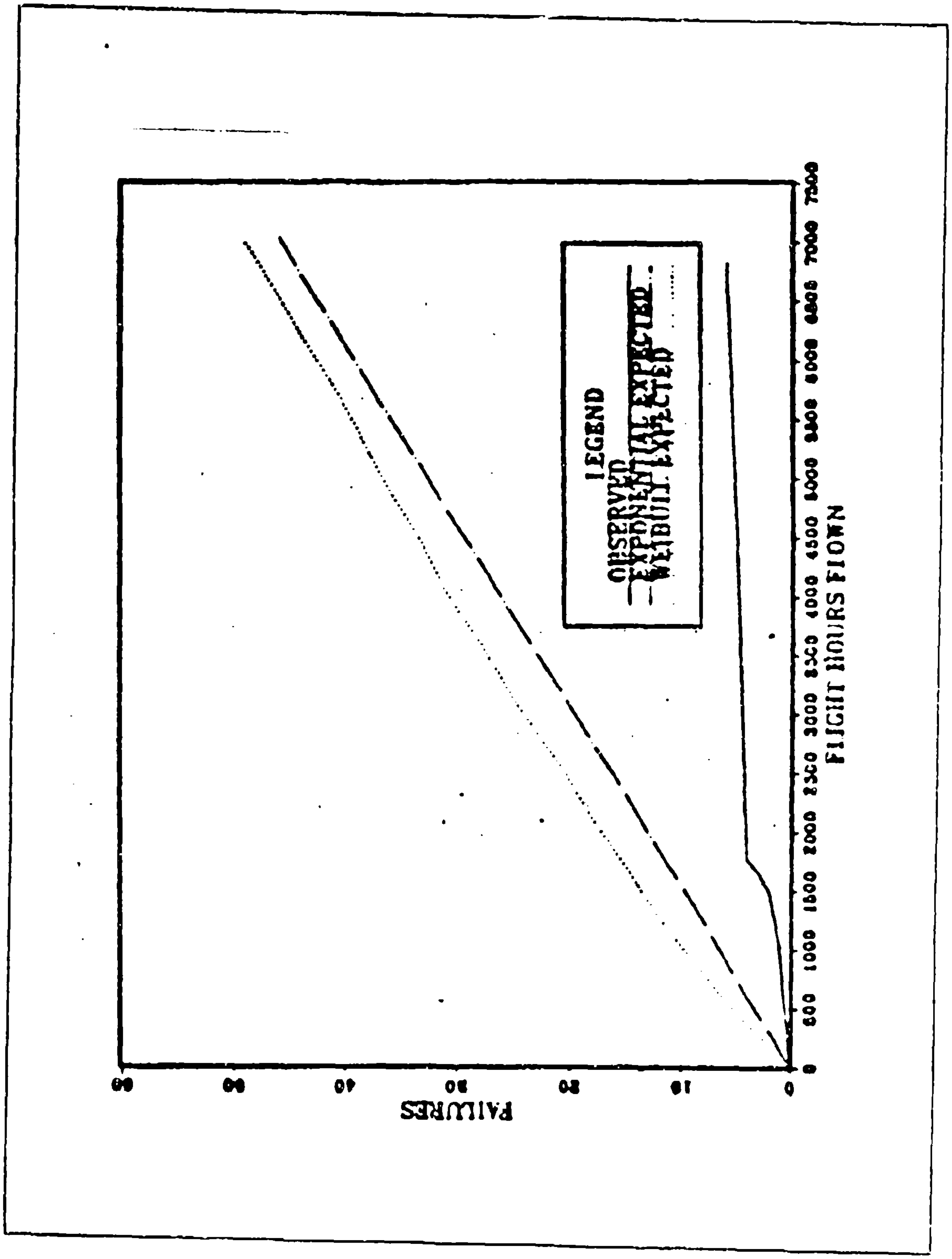

Figure I.11 Flight Hour Models: Validation Set WUC:4622100.

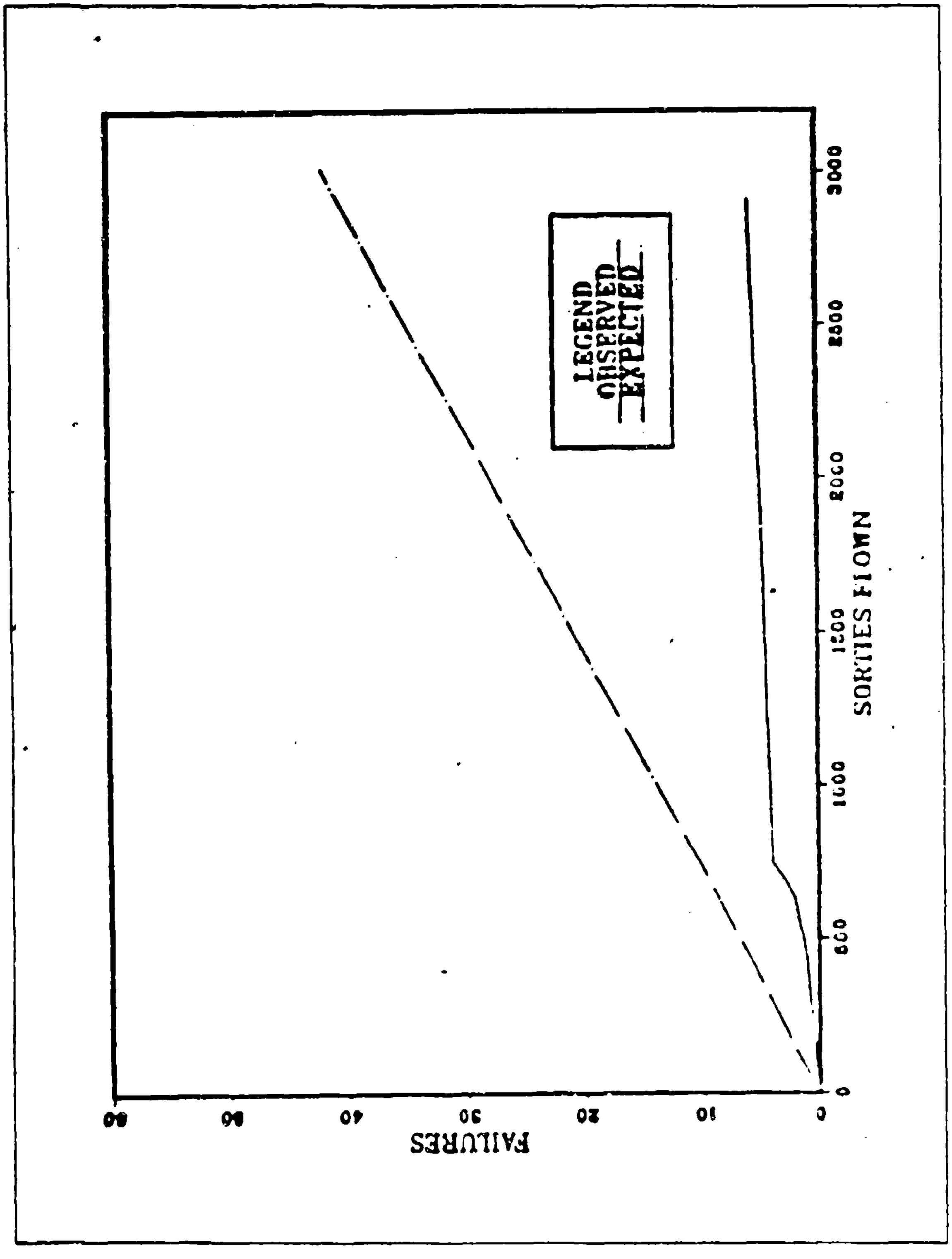

Figure I.12 Geometric Model: Validation Set WUC:4622100.

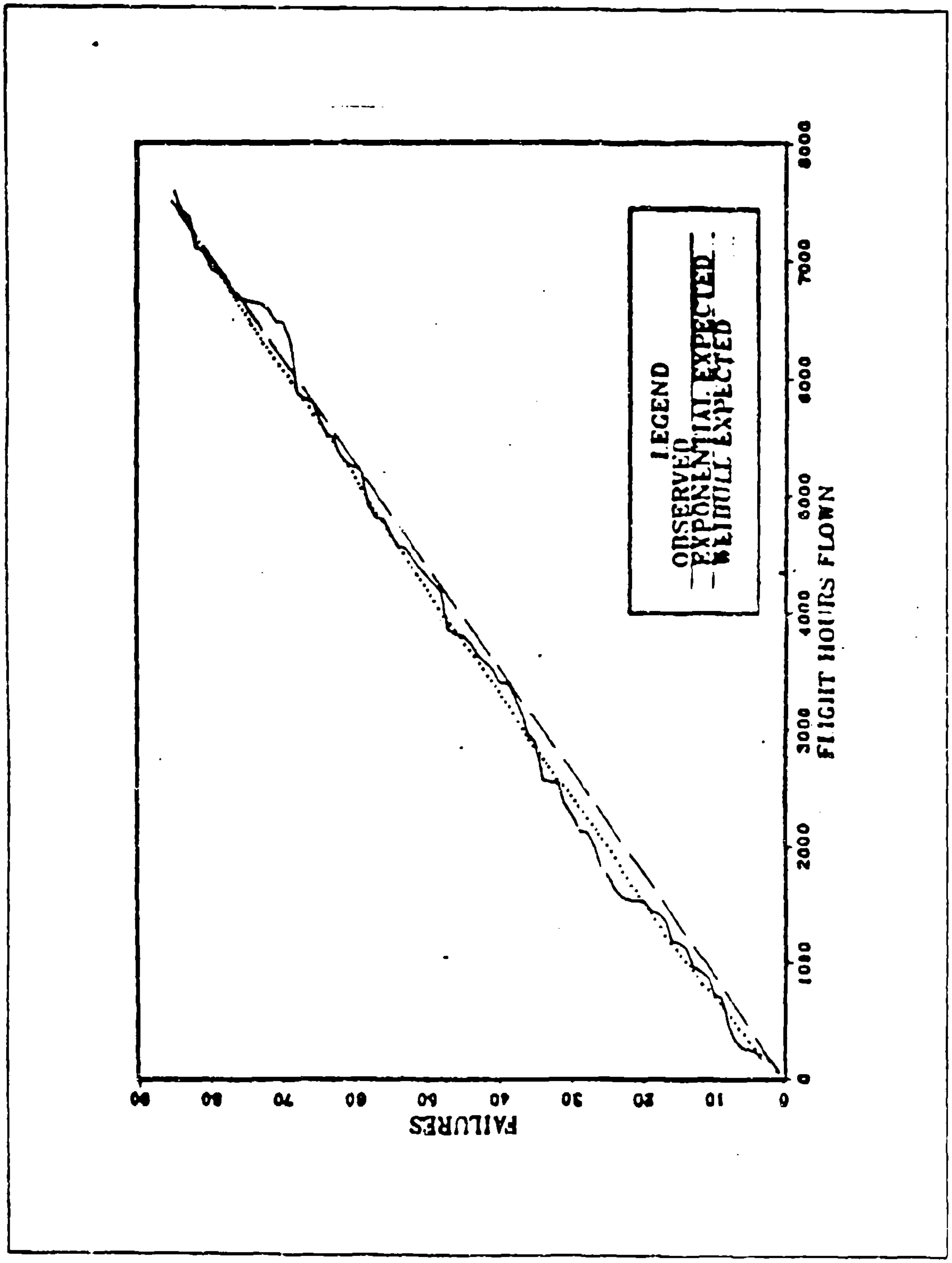

Figure I.13 Flight Hour Models: Model Set WUC:56X2500.

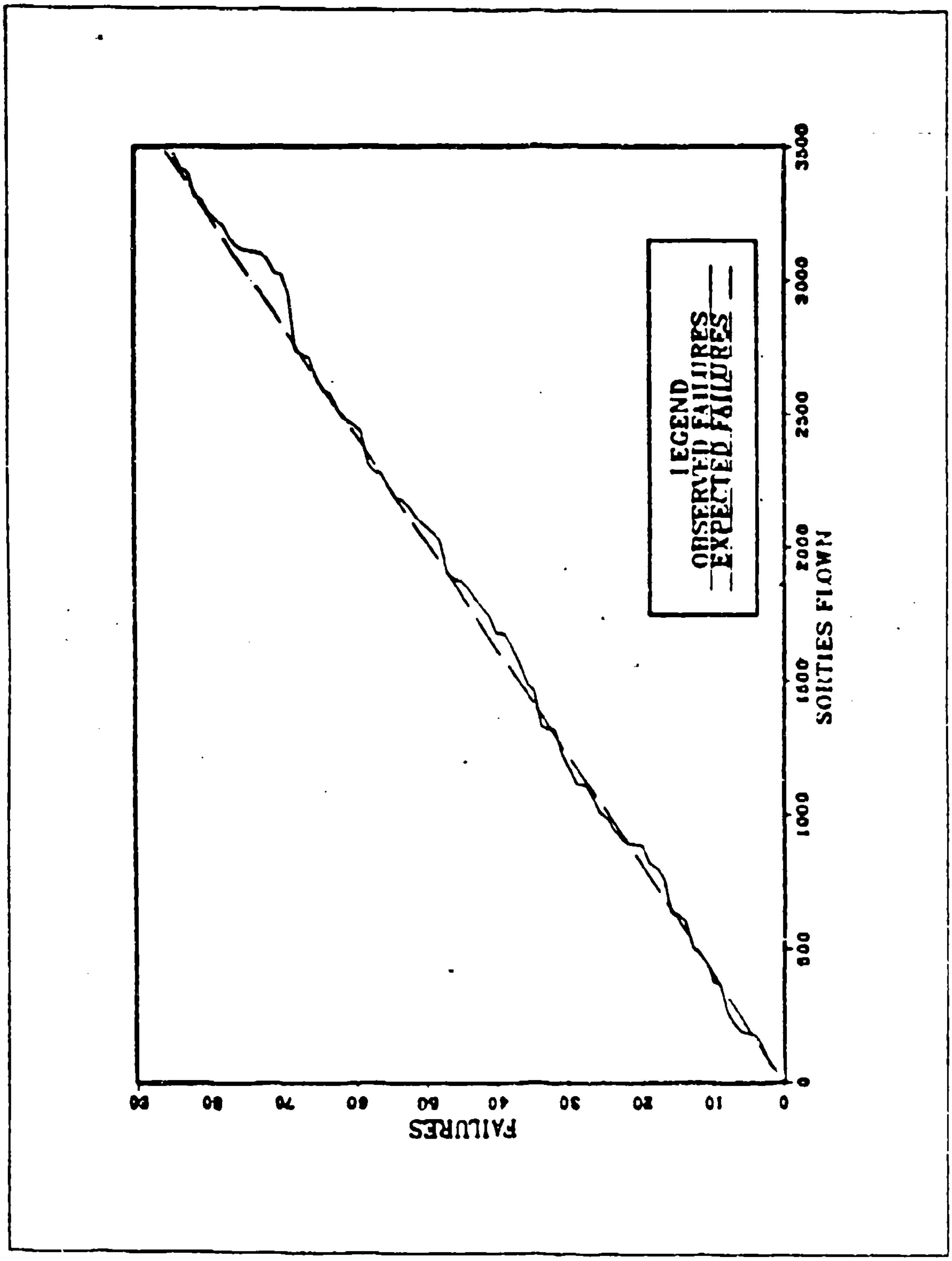

Figure I.14 Geometric Model: Model Set WUC:56X2500.

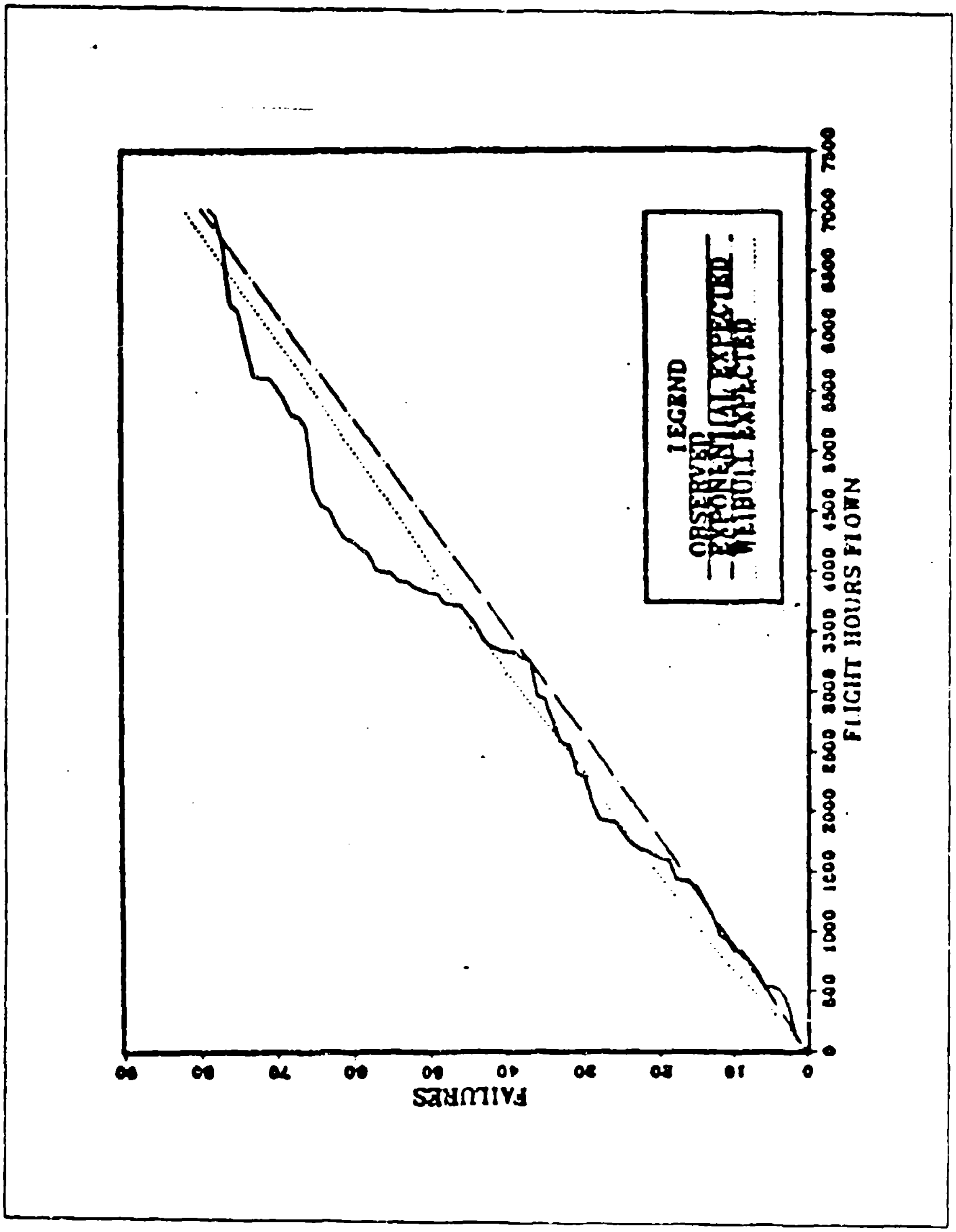

Figure I.15 Flight Hour Models: Validation Set WUC:56X2500.

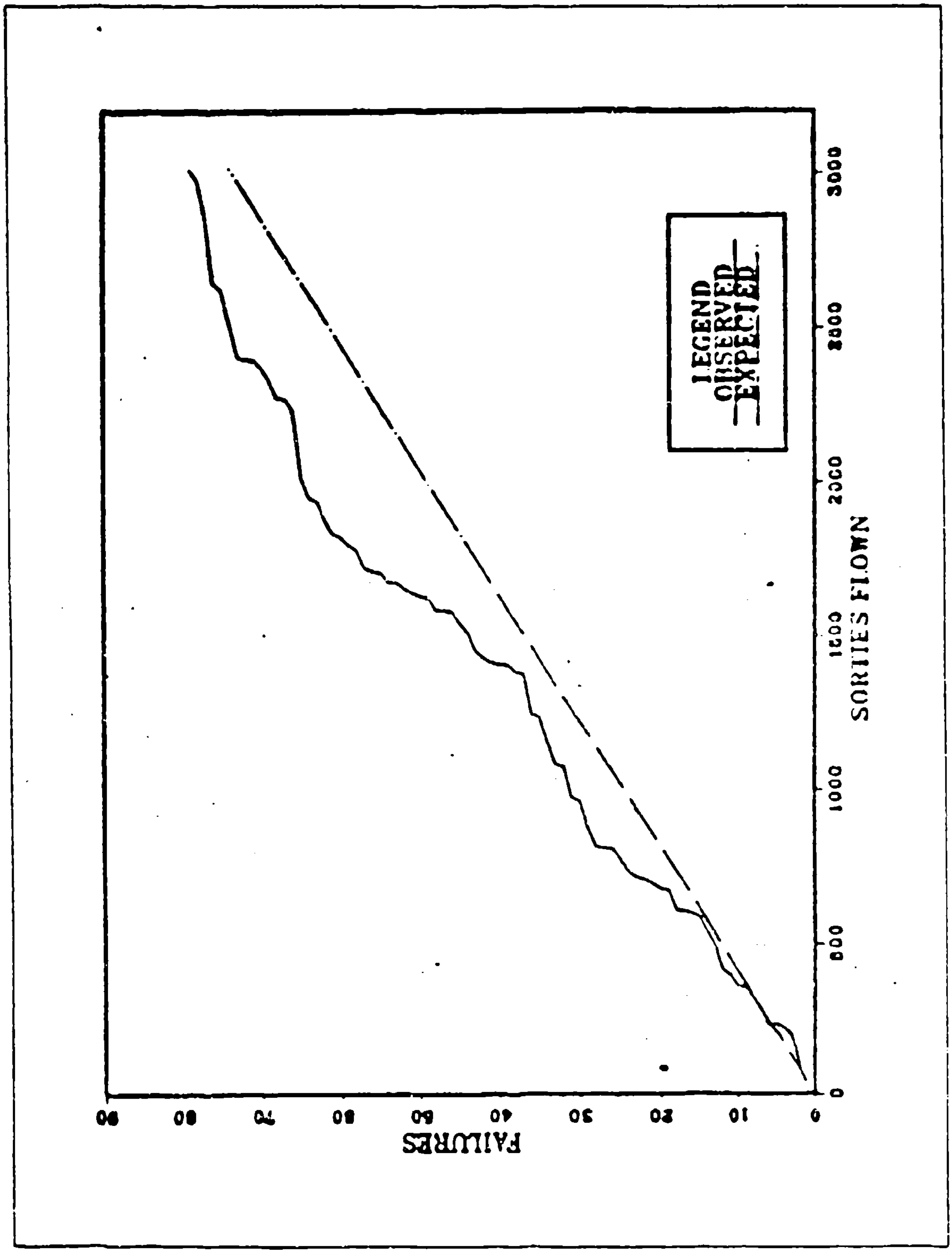

Figure I.16 Geometric Model: Validation Set WUC:56X2500.

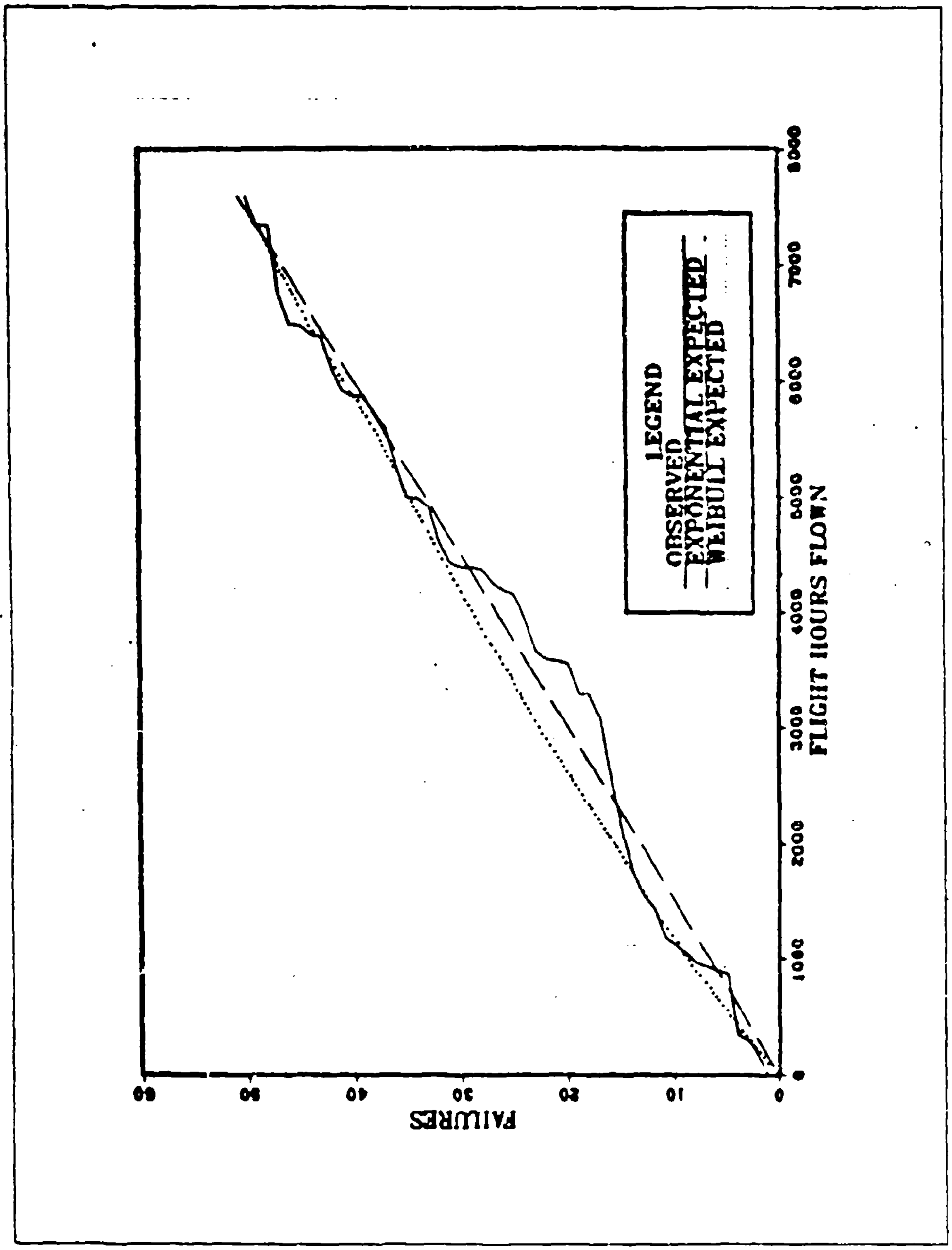

Figure I.17 Flight Hour Models: Model Set WUC:632Z100.

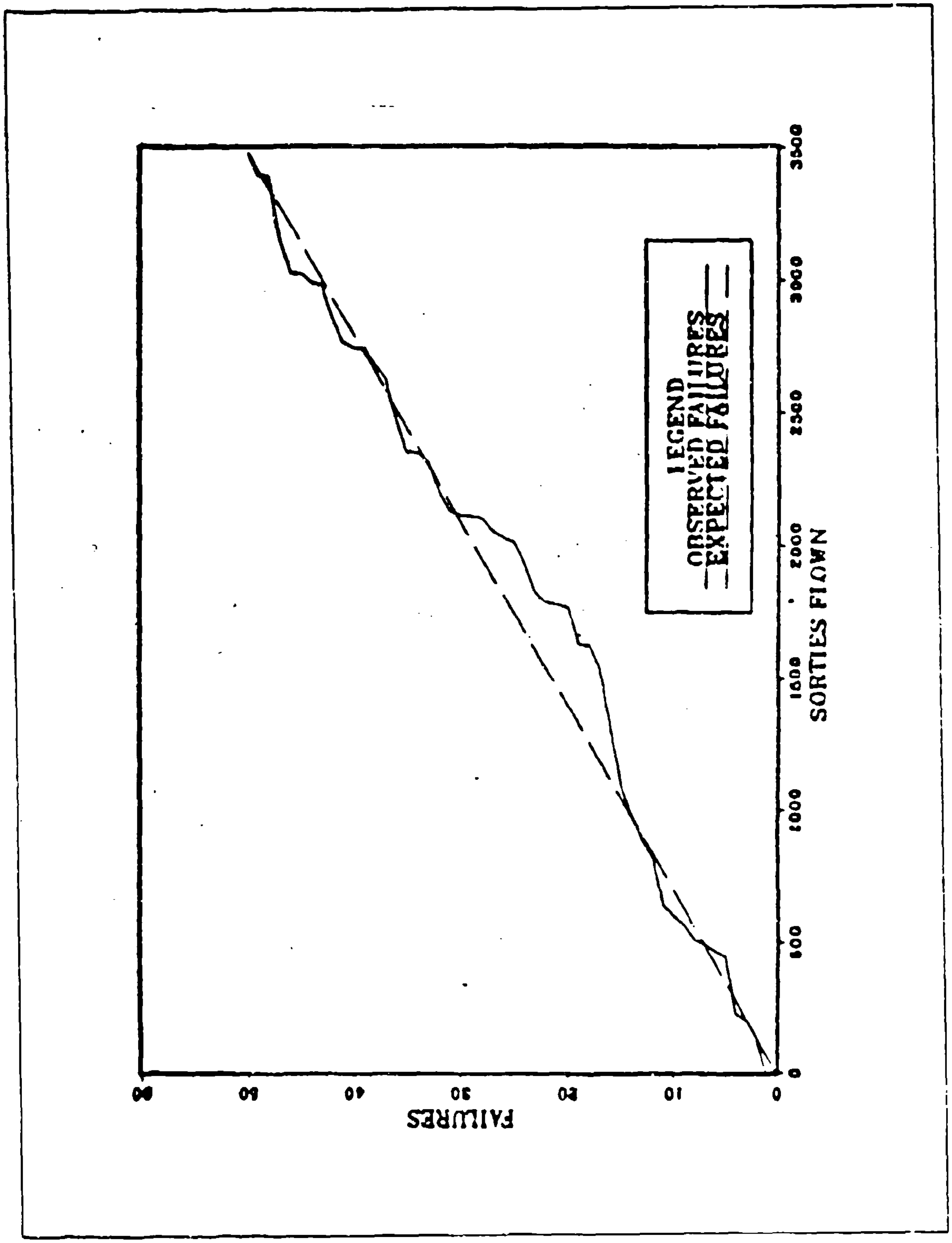

Figure I.18 Geometric Model: Model Set WUC:632Z100.

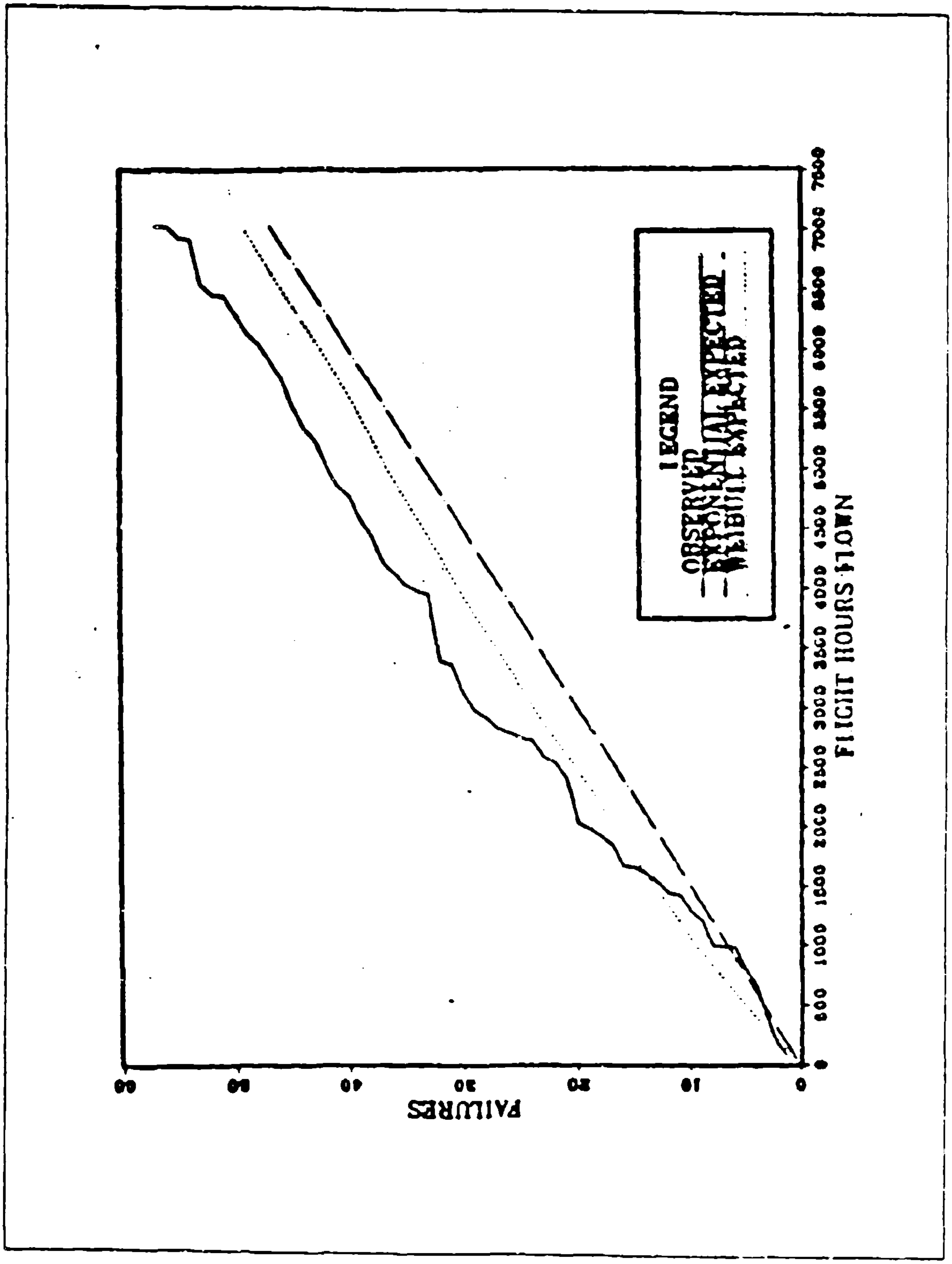

Figure I.19 Flight Hour Models: Validation Set WUC:632Z100.

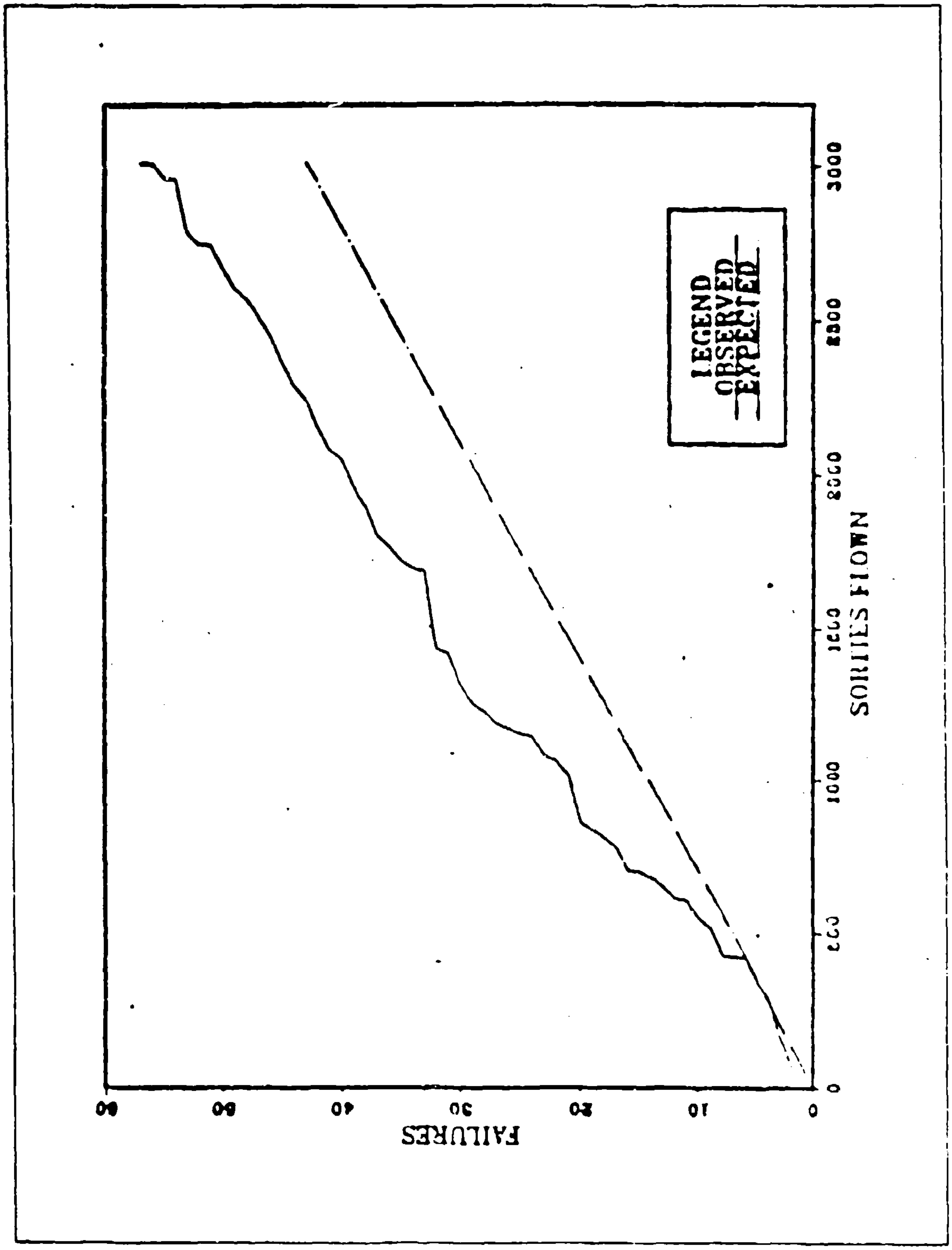

Figure 1.20 Geometric Model: Validation Set WUC:632Z100.

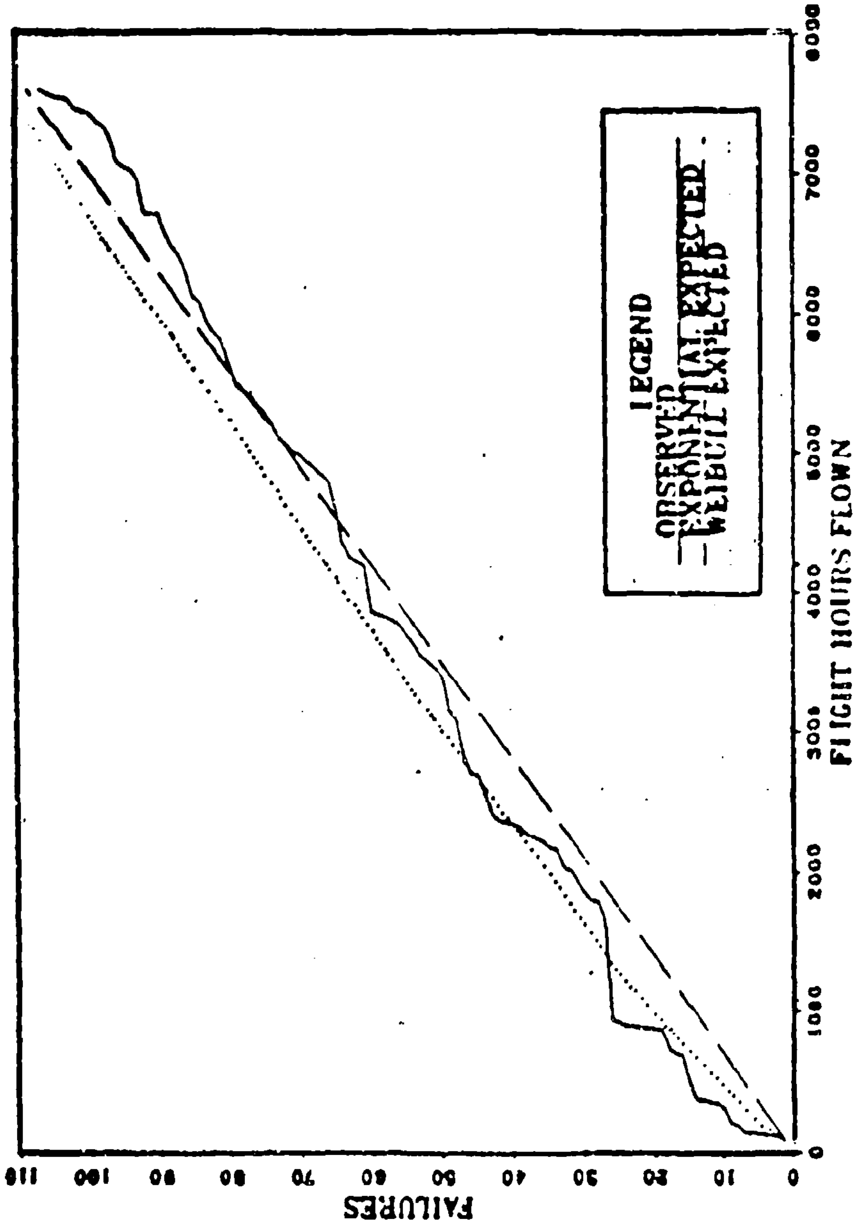

Figure I.21 Flight Hour Models: Model Set WUC:74A1500.

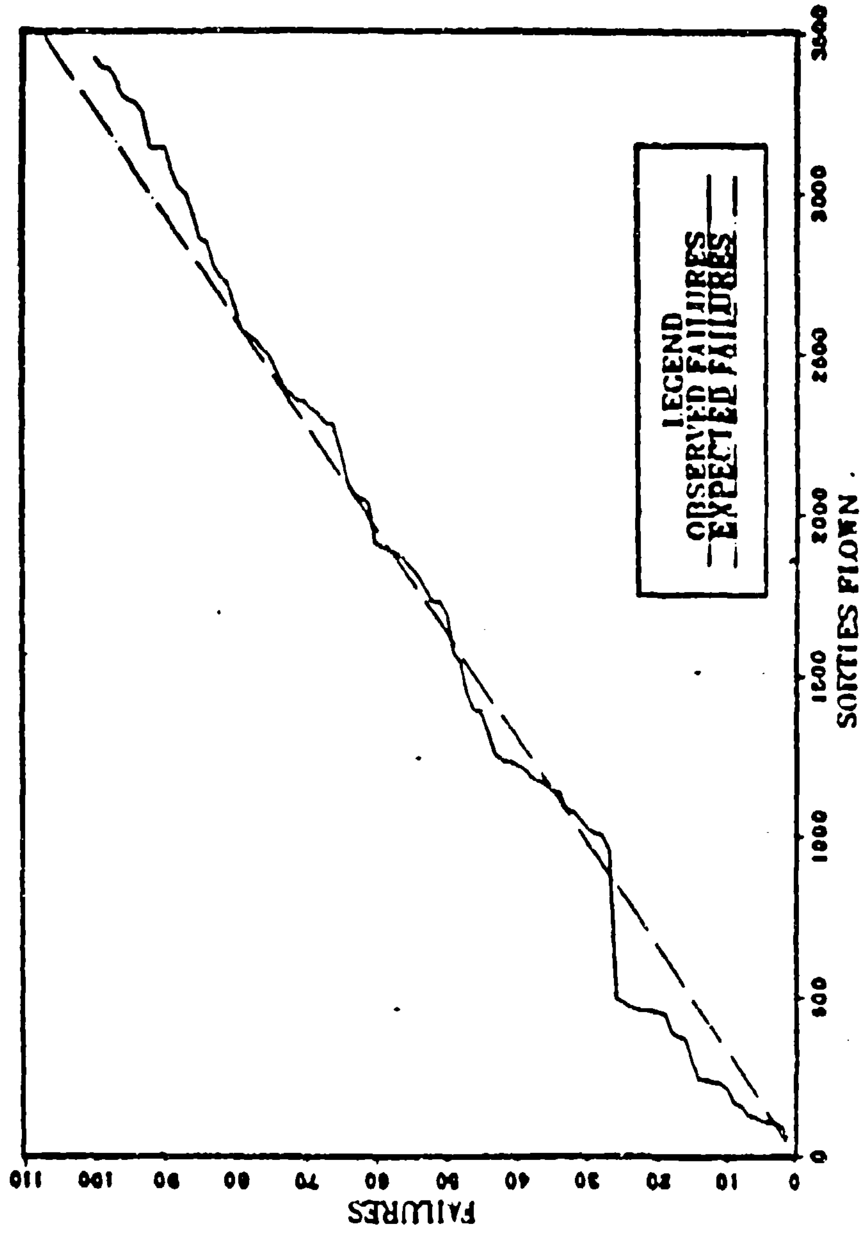

Figure 1.22 Geometric Model: Model Set WUC:74A1500.

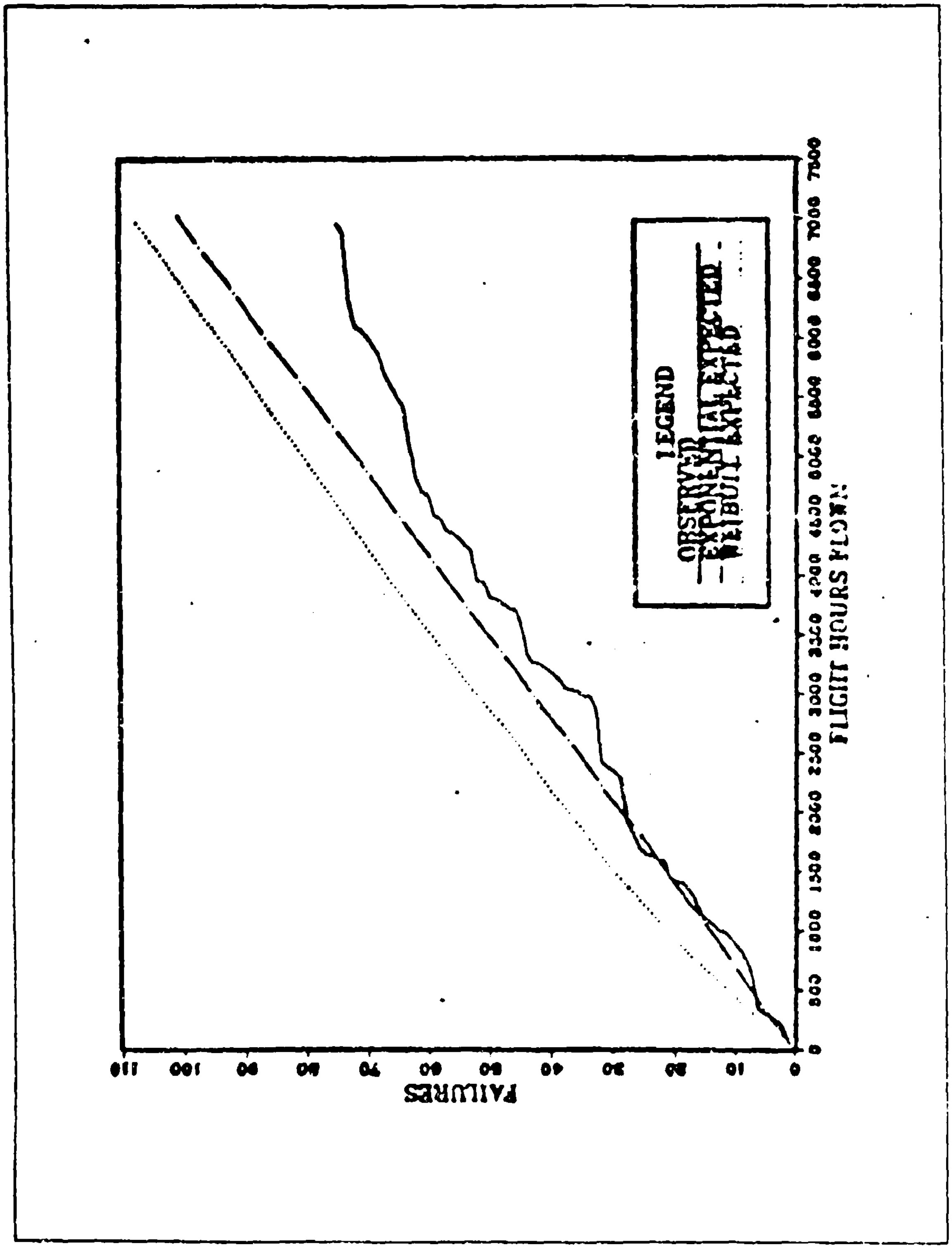

Figure I.23 Flight Hour Models: Validation Set WUC:74A1500.

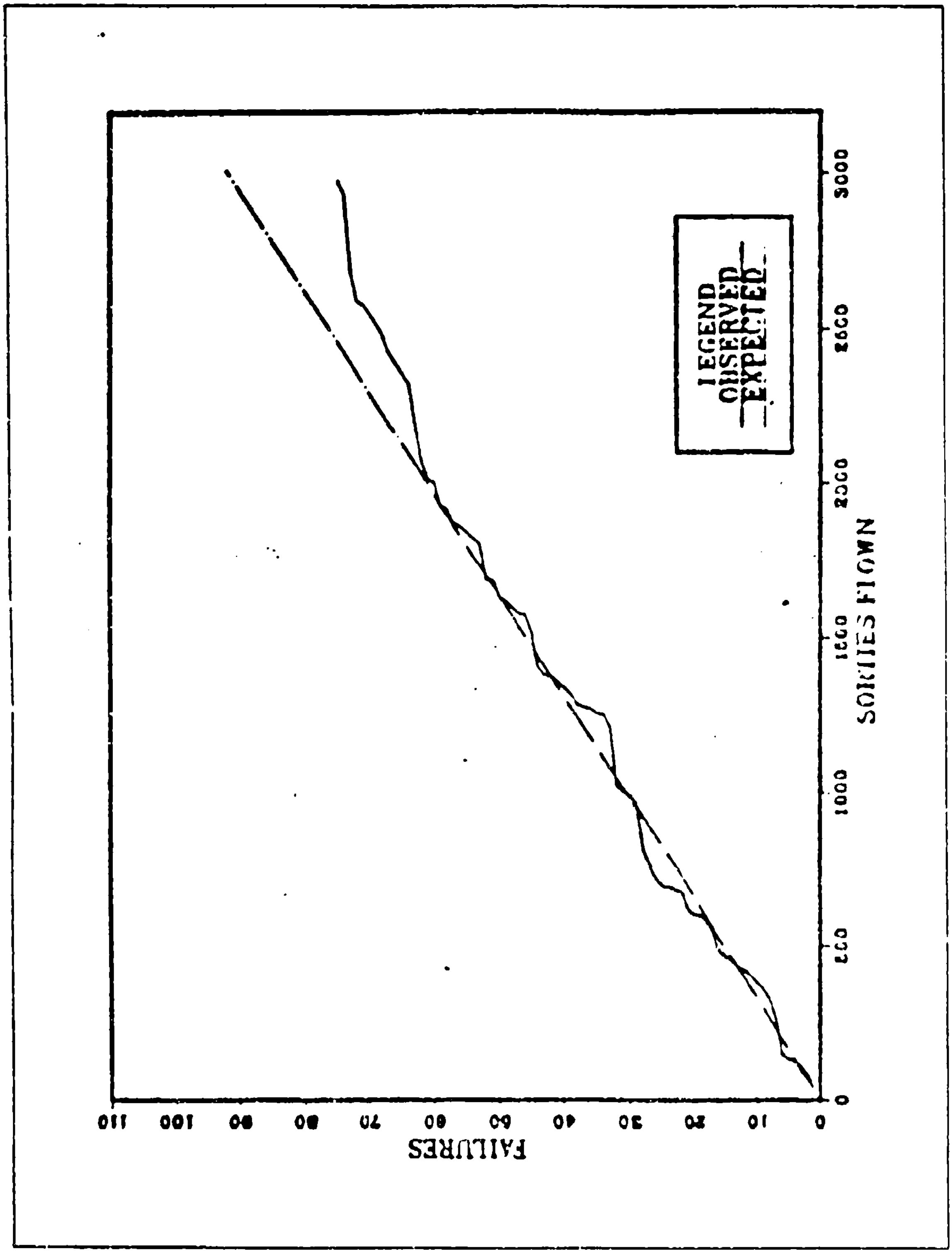

Figure 1.24 Geometric Model: Validation Set WUC:74A1500.

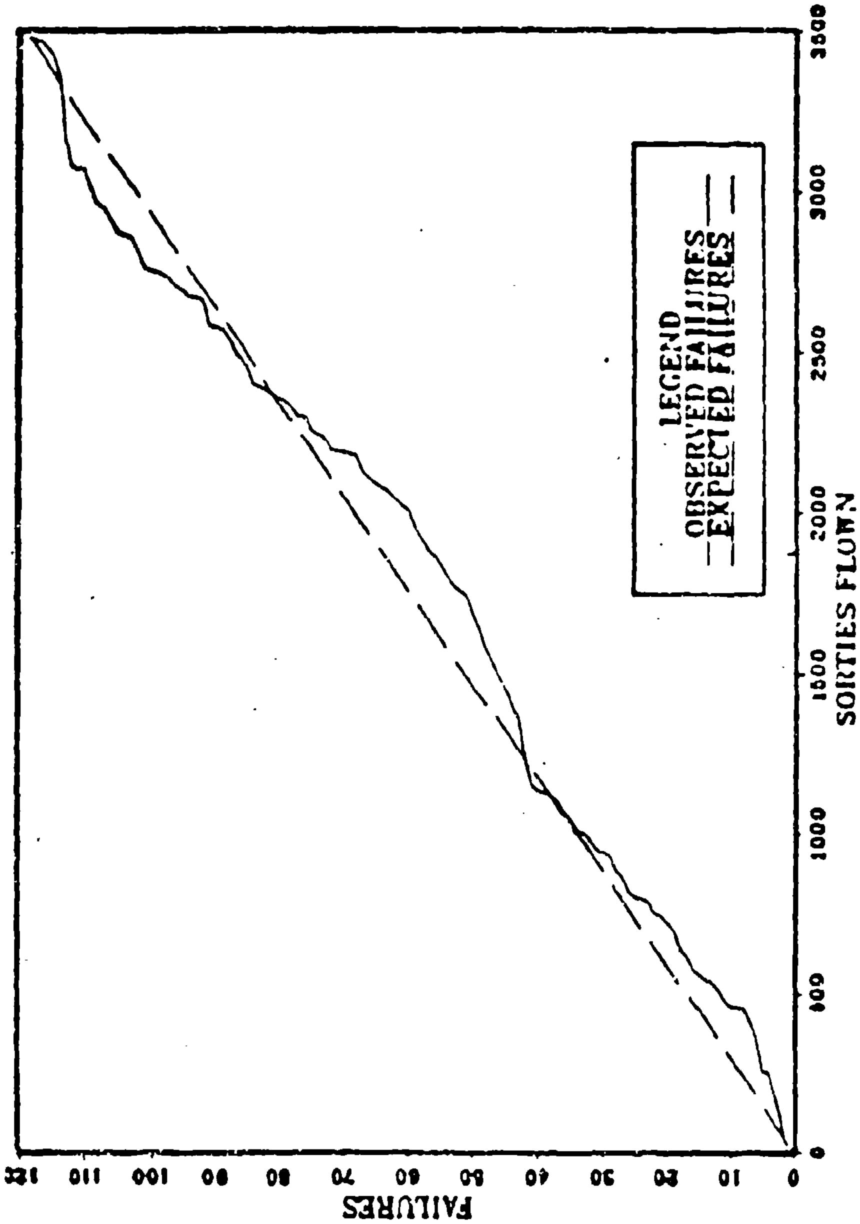

Figure I.25 Geometric Model: Model Set WUC:74A5M00.

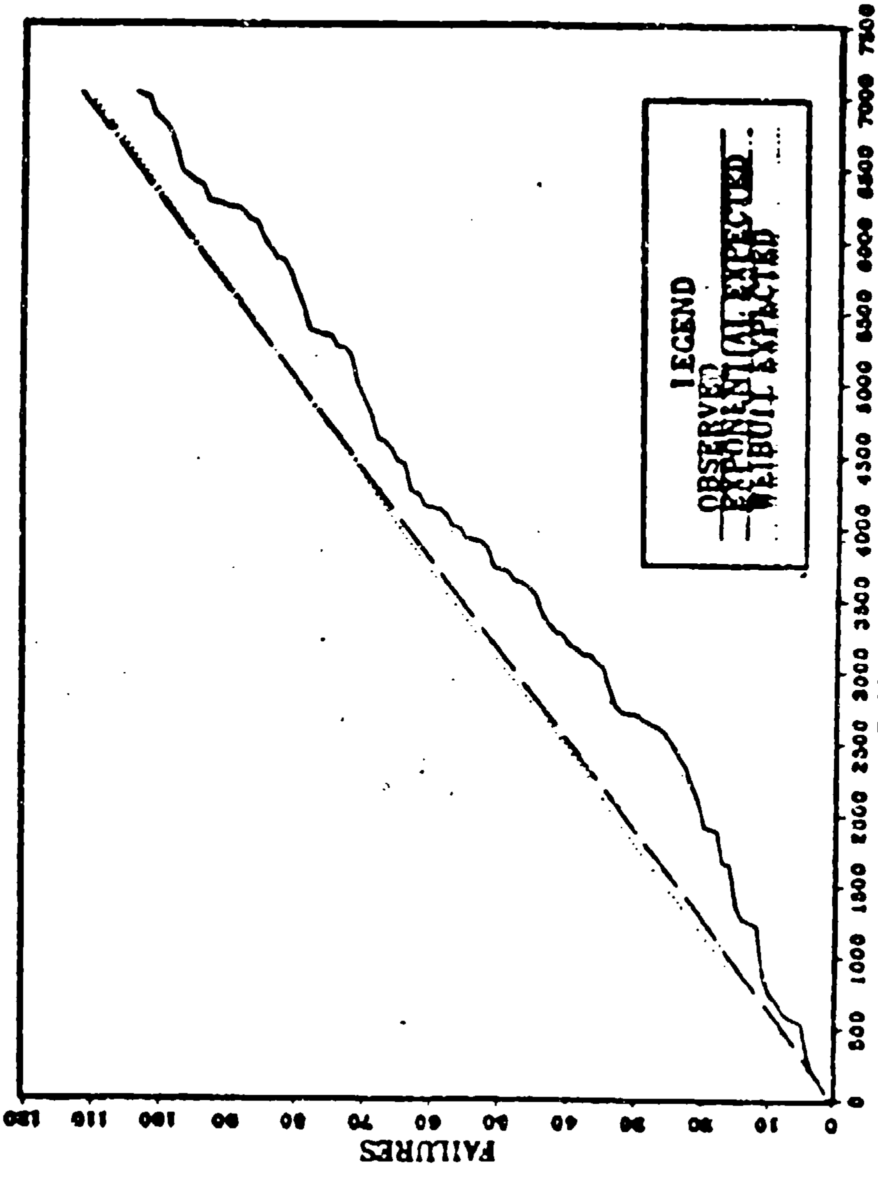

Figure 1.26 Flight Hour Models: Validation Set WUC:74A5M00.

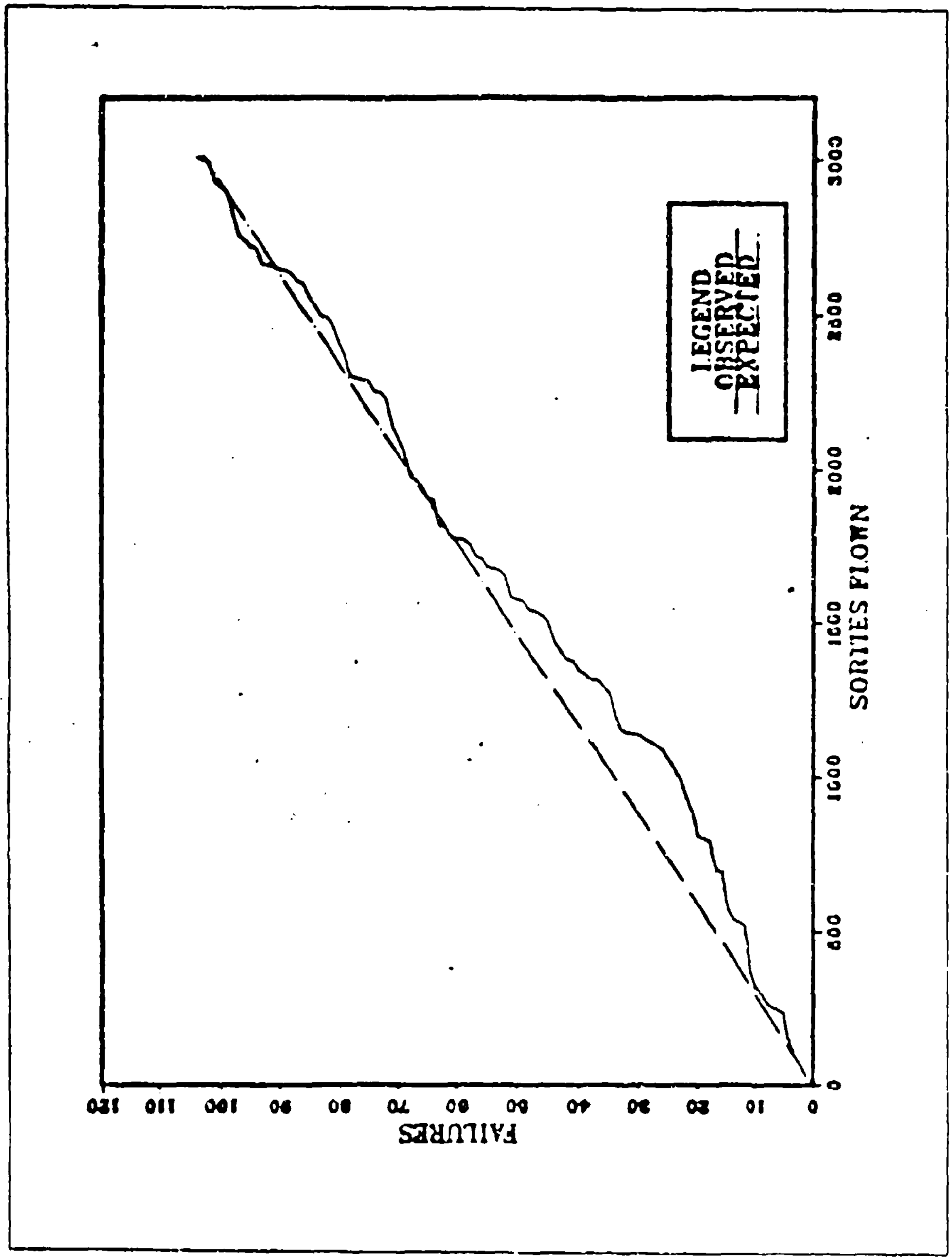

Figure 1.27 Geometric Model: Validation Set WUC:74A5M00.

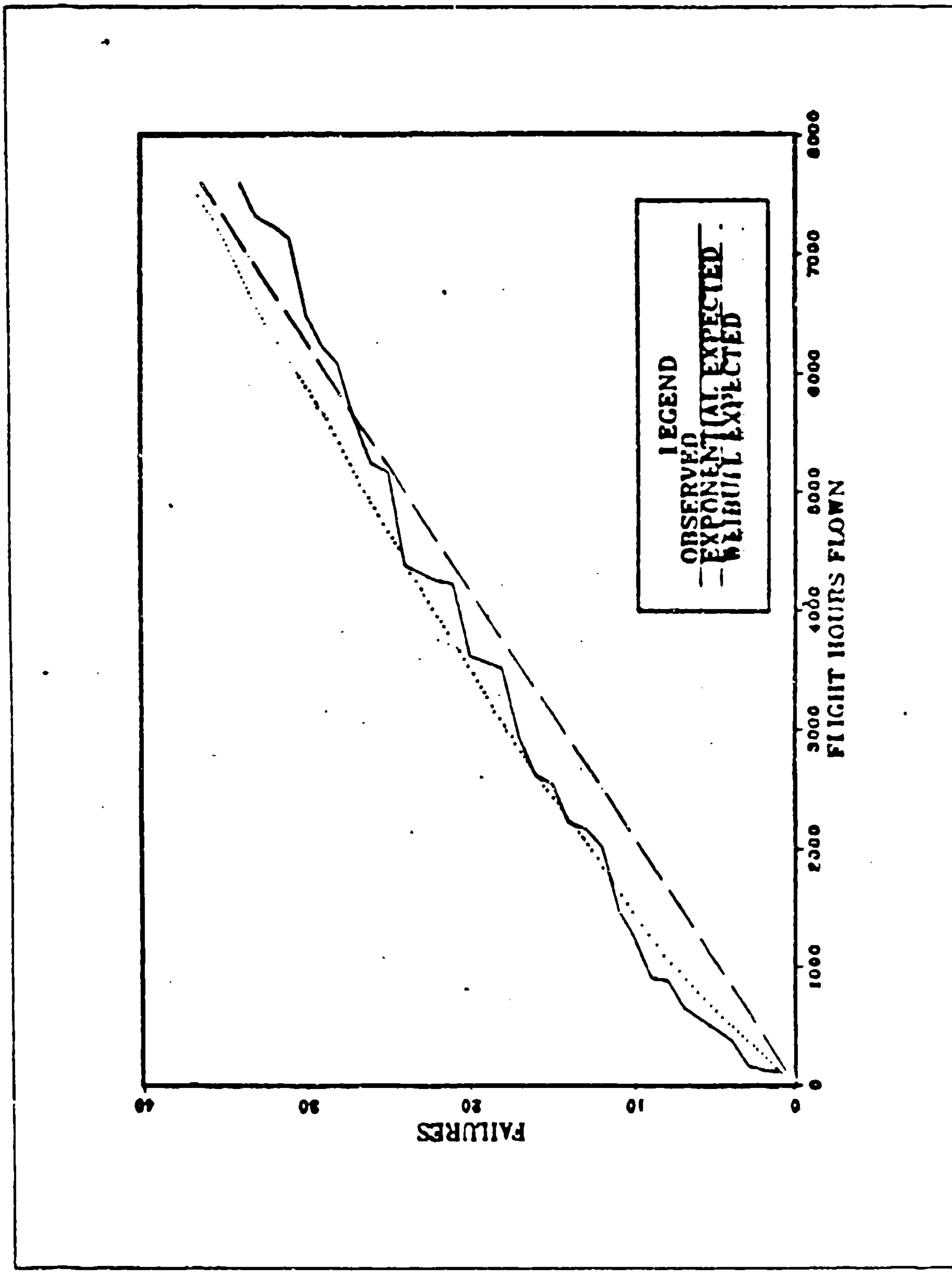

Figure I.28 Flight Hour Models: Model Set WUC:5772200.

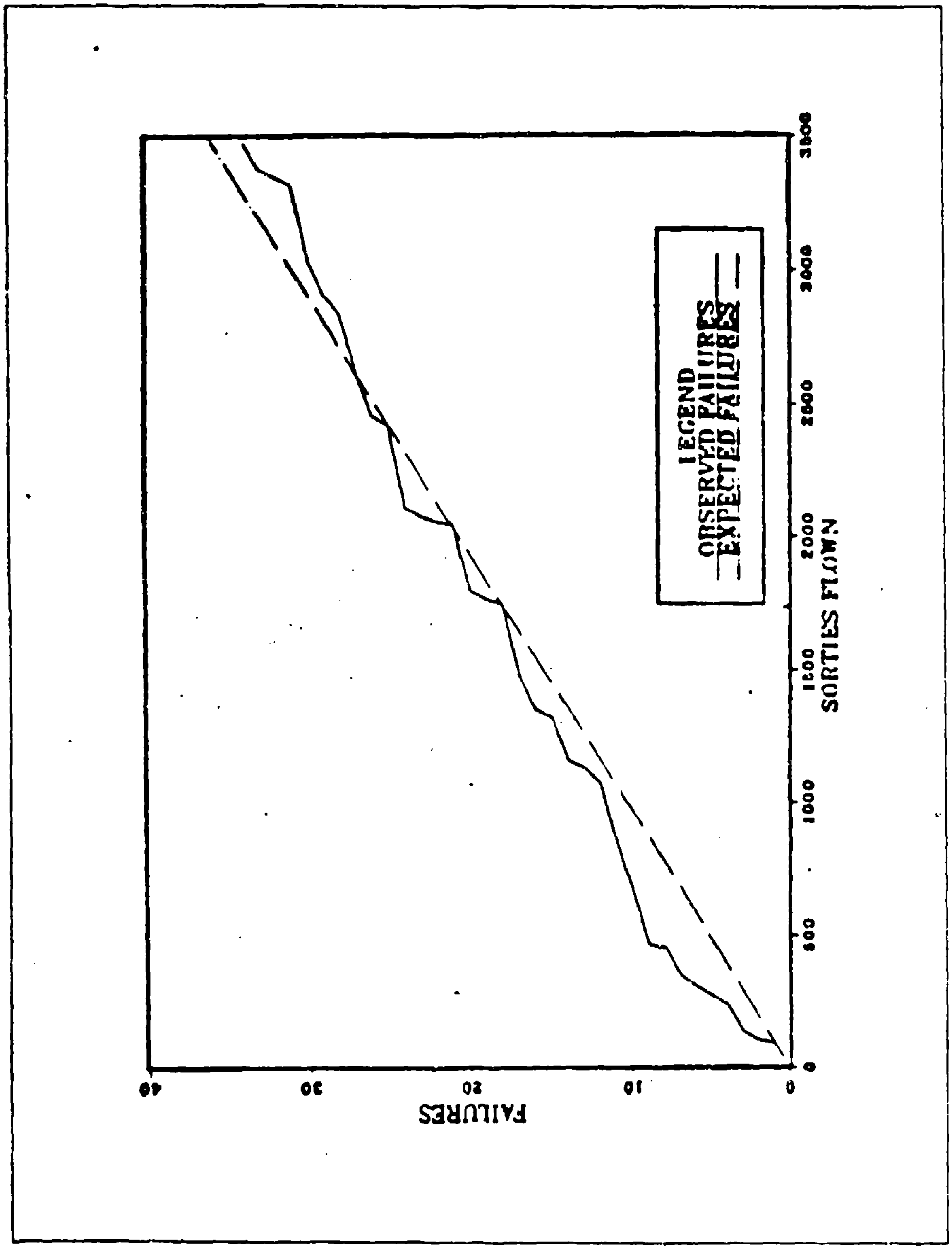

Figure I.29 Geometric Model: Model Set WUC:5772200.

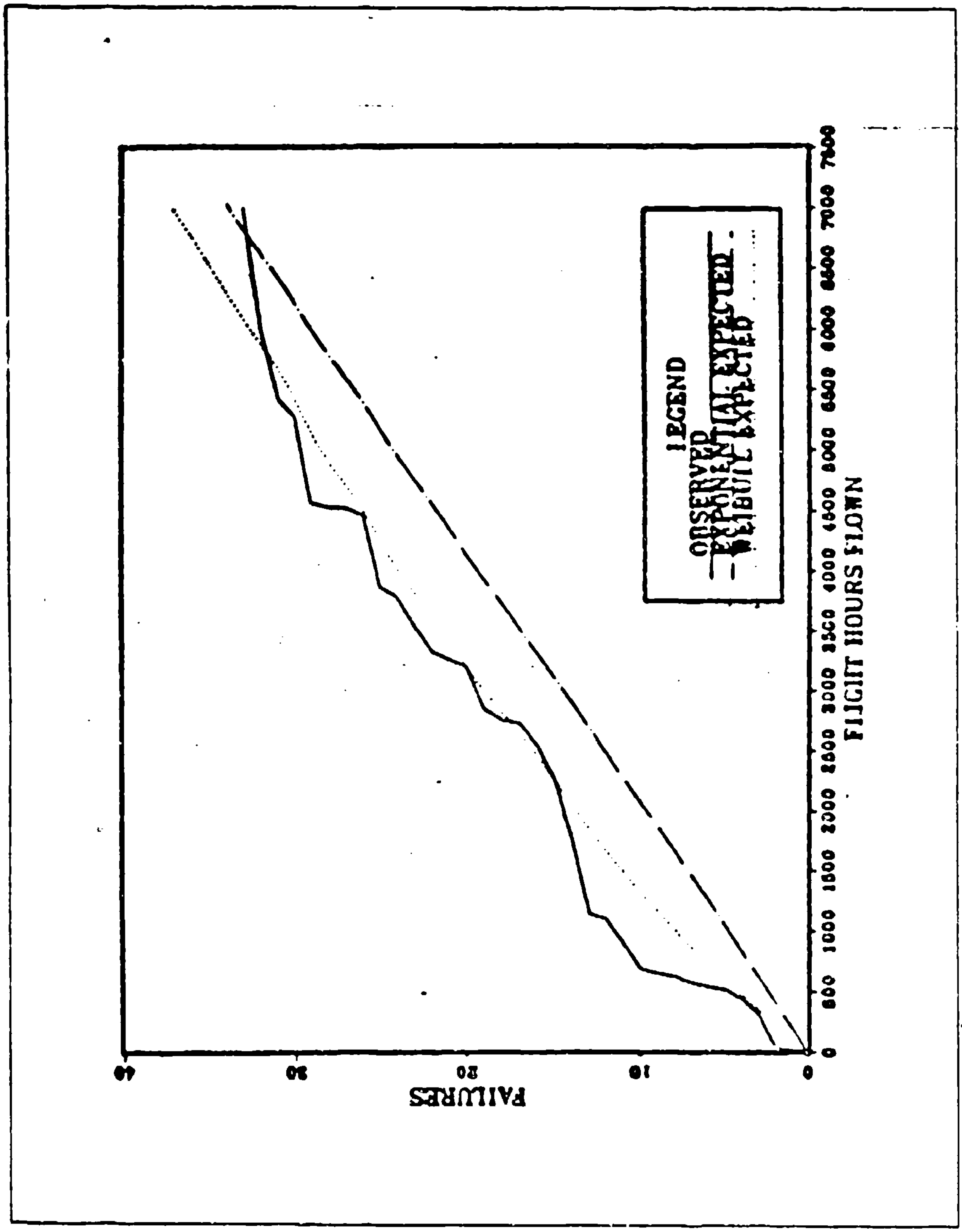

Figure I.30 Flight Hour Models: Validation Set WUC:5772200.

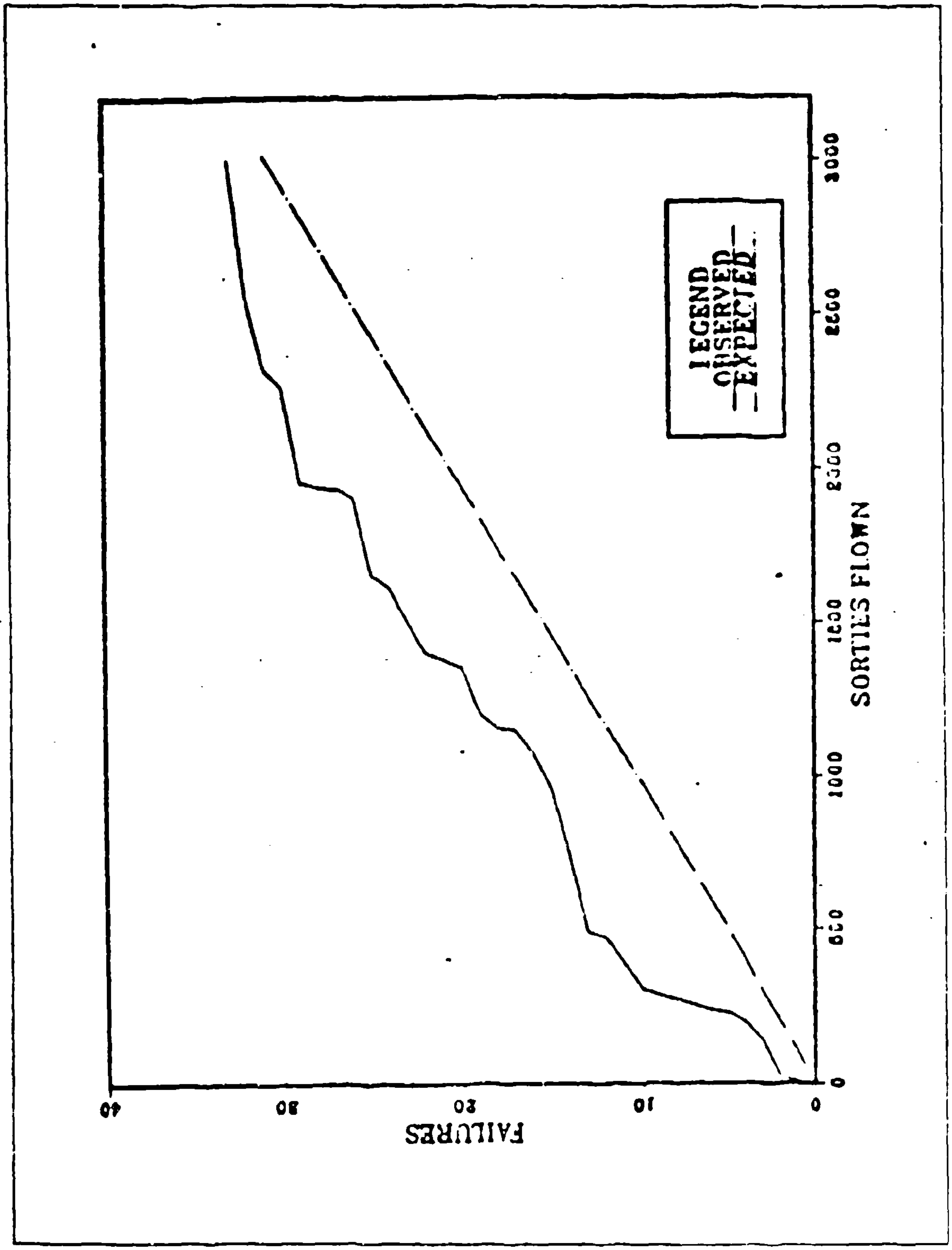

Figure I.31 Geometric Model: Validation Set WUC:5772200.

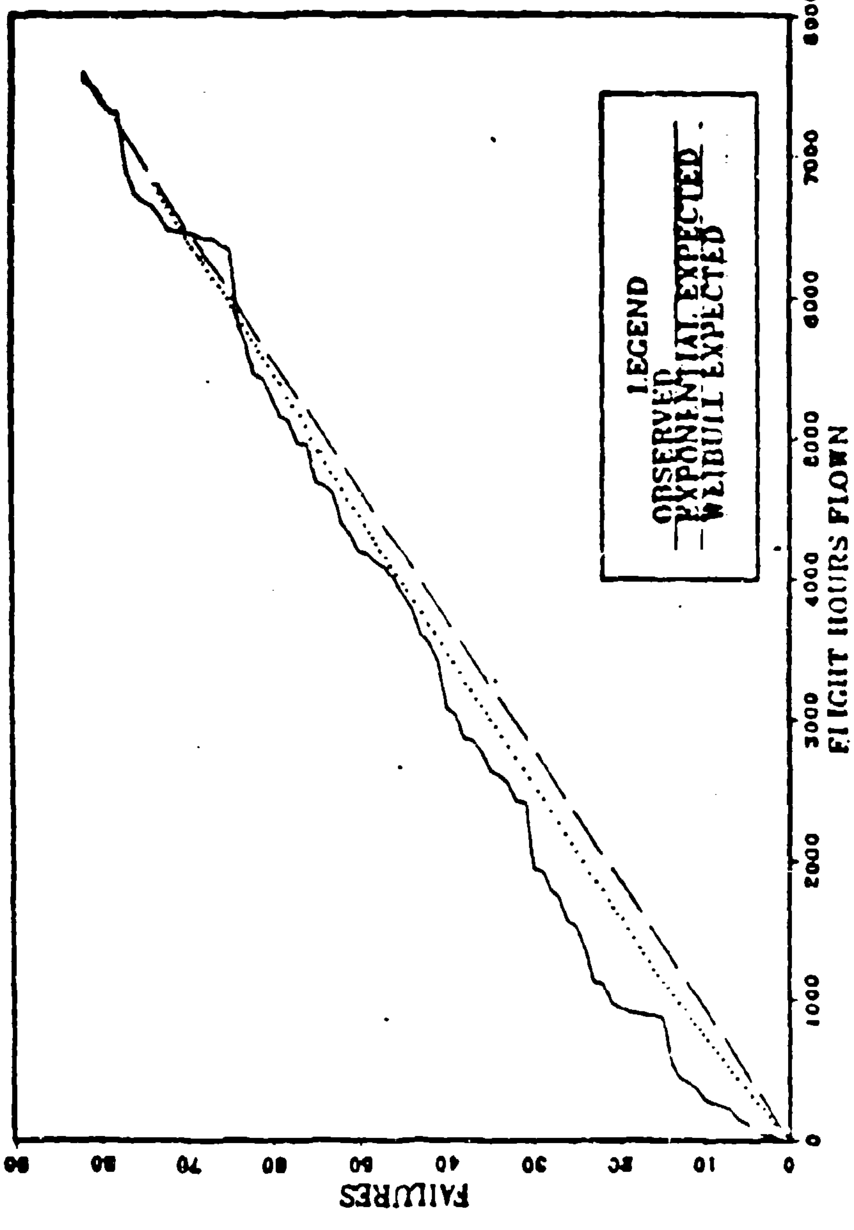

Figure I.32 Flight Hour Models: Model Set WUC:6918100.

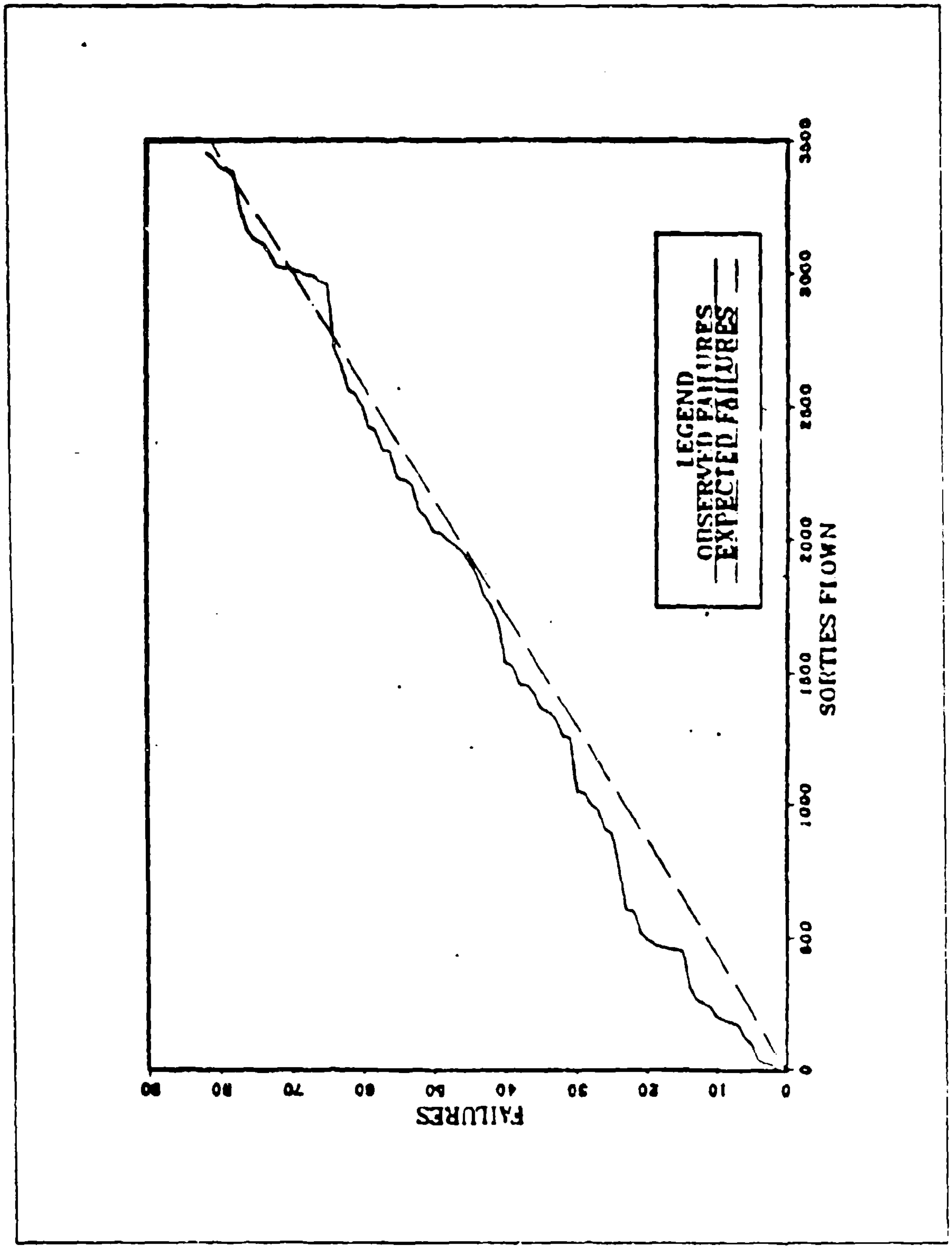

Figure I.33 Geometric Model: Model Set WUC:6918100.

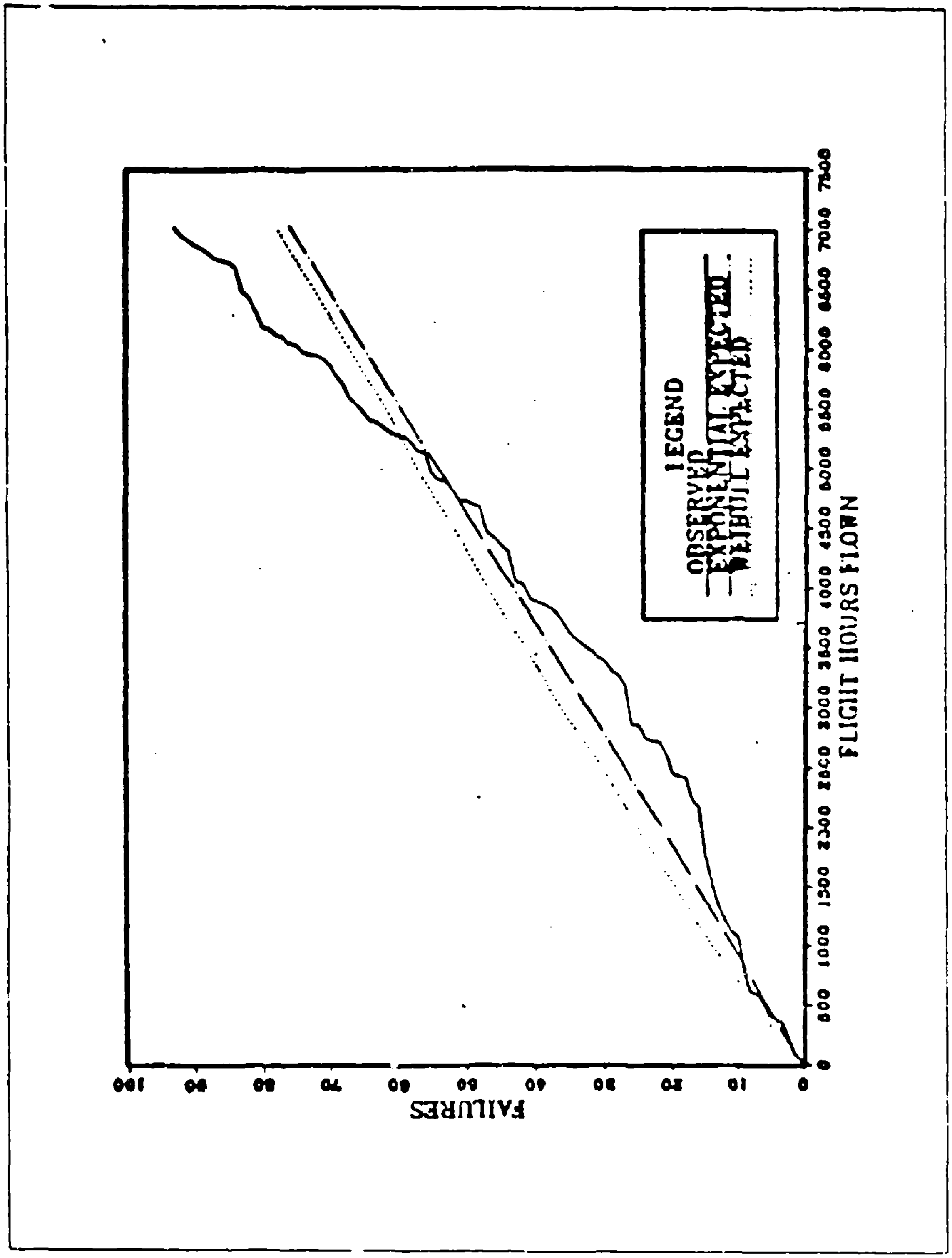

Figure I.34 Flight Hour Models: Validation Set WUC:6918100.

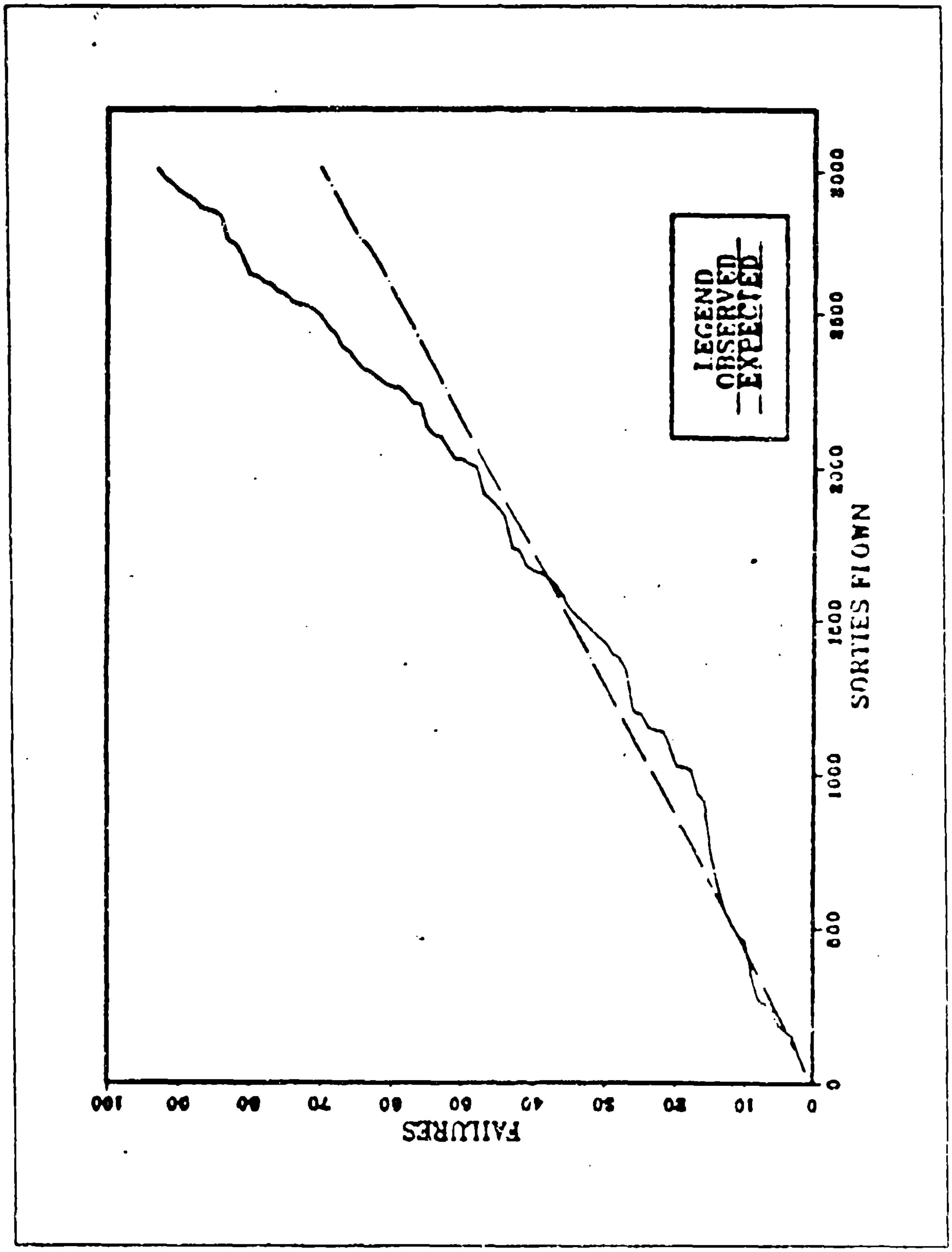

Figure I.35 Geometric Model: Validation Set WUC:6918100.

LIST OF REFERENCES

1. Garrison, Peter, *CV:Carrier Aviation*, p. 87, Presidio Press, 1980.

2. Rand Corporation R-2853-NAVY, *Carrier Air Based Logistics Study: Integrated Summary*, by T.F. Lippiatt, and others, January 1982.

3. Center for Naval Analyses CNS 1180, *Aviation Parts Allowance Policy*, by Peter Evanovich, and others, August 1984.

4. Mitchell, M. L., *A Retail Inventory Model for Naval Aviation Repairable Items*, Masters Thesis, Naval Postgraduate School, Monterey, California, March 1983.

5. Center for Naval Analyses CRM 86-41, *Distinguishing the Effects on Failures of Changes in Sortie Rate and Sortie Length*, by Robert A. Levy, pp. 1-5, March 1986.

6. Center for Naval Analyses, *The Relationship Between Failures and Sortie Length*, by Robert A. Levy and others, 1986.

7. Chambers, John M., and others, *Graphical Methods for Data Analysis*, Duxbury Press, 1983.

8. Rand Corporation N-2165-AF, *Estimating Wartime Support Resource Requirements: Statistical and Related Policy Issues*, by B. Embry, July 1984.

9. Taylor, H. M., and Karlin, S., *An Introduction to Stochastic Modeling*, Academic Press, Inc., 1984.

10. Larson, Harold J., *Introduction to Probability Theory and Statistical Inference*, Third Edition, John Wiley & Sons, Inc., 1982.

11. Center for Naval Analyses 1105-75, *Determining the Failure Rate When Failure Times Are Not Known Exactly*, by W.J. Heintzelman, 12 August 1975.

12. Reklaitis, G.V., and others, *Engineering Optimization: Methods and Applications*, John Wiley and Sons, Inc., 1983.

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